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Design of Controller (Compensator) For Position-Controlled System with DC Motor

Emmanuel E. Effiong, Peter Onwe, Ekpenyong N. Oku, Yaabari Naenwi

Abstract— **This paper covers the design of Lead Compensator for closed loop position control system using field control separately excited direct current motor (SEDCM). The closed loop position control system was developed in MATLAB / SIMULINK software. The aim of compensator design is to improve the transient response of the dc motor shaft (motor) position to step field winding voltage using Bode diagram design technique. The model of the position control system was tested using MATLAB / SIMULINK. The design specification of the system was evaluated and from the simulation results. The compensated system satisfies the design specification and this shows that compensators can be designed with ease using frequency response design method such as Bode diagram and Nichol chart design approach.**

Index Terms— **SEDC motor, Lead Compensator, Frequency response, Bode diagram**

I. Introduction

Direct current (DC) motors are extensively used in industrial and domestic applications due to its good characteristics such as high starting and accelerating torque, high response performance and it is easy to be controlled. DC motor are widely used in steel rolling mills, cutting tools, robotics manipulators, overhead cranes, electric train, home appliance speed and position applications etc. [1]. Separately excited DC motor (SEDCM) are suitable for adjustable speed application, the voltage is supplied directly to the field winding of the motor. There are three speed control techniques commonly used for Dc motor: there are: (i) Field resistance control (ii) Armature resistance control and (iii) Armature voltage control. In Armature controlled dc drive of SEDCM, the speed is directly proportional to the applied armature voltage of the dc motor. Hence, speed of the motor can be controlled by adjustment of terminal voltage while keeping the field voltage constant [2] In field (flux) control strategy, SEDCM is controlled by adjustment of field resistance since the flux is produced by field current. The speed of the motor increases as flux or field current decreases, that is speed is inversely proportional to flux per pole or field current as the armature current is kept constant. This variation of field current is achieved in shunt typed-dc motor by the use of shunt variable resistor known as shunt field regulator. In series-types dc motor, the field current is controlled by the use of diverter (resistor) or trapped field control [3]. Dc motor is a set of rotatory electrical machine which convert direct current electrical energy into mechanical energy. A controller takes the input (voltage or current) to control the output (speed, position of the motor shaft) regardless of the load. The speed control is achieved by using operational amplifier (OPAMP) circuit design for proportional integral control as this produces quick, smooth motor response to the input as speed of the motor is regulated [4]. Dc motor converts electrical power to mechanical output (shaft rotation). However, no system is ideal and therefore, there exist some power consuming elements. These causes a decrease in efficiency and performance of the system, such consuming elements include resistance, inductance, inertia, friction etc. It becomes necessary to use controller (compensator) with the system (dc motor). this helps the motor to produce desired response at the output (speed, position of shaft) with accuracy and stability [5]. Feedback control system with compensator is important technique that is widely used in process industries, as this compensator provides corrective actions as soon as controlled variables deviates from the set point regardless of the disturbance type and source of disturbance. Compensators are corrective sub-system that is added into system to compensate for the deficiency in performance of the plant, when the plant (system) fails to provide desires response of system specification. Common type of compensators that are effective include: phase-lead, phase-lag, phase –lead– lag compensators which are essential for various frequency–based design. Such design method is based on Bode plots and alternative method called Root locus design [6]. In order to obtain the desired performance of control system, compensating networks are needed, These compensators influence the following disciplines such as Robotics, satellite control, automobile diagnostic and laser frequency stabilization. For compensator design using frequency method (Bode plot and Nichol chart technique), we specify the performance specifications, that is frequency domain specifications in indirect manner such as gain margin, phase margin, Bandwidth, and also transient –response performance specification (time–domain specification) which include settling time, peak overshoot, rise time and peak time. The specifications can be conveniently met in Bode diagram design approach or frequency domain design which is simple and straight forward. After design, the performance indices are checked to see whether the designed system specification are met, If it does not, then the compensator must be modified or redesigned repeatedly until a satisfactory performance is obtained [5]. In this paper, the design specifications based on time domain approach is adopted for position control system with dc motor. The

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Emmanuel E. Effiong, Department of Electrical/Electronic Engineering, University of Cross River, Calabar, Cross River State, Nigeria.
Peter Onwe, Department of Electrical/Electronic Engineering, University of Calabar, Cross River State, Nigeria
Ekpenyong N. Oku, Department of Electrical/Electronic Engineering, University of Cross River, Calabar, Cross River State, Nigeria.
Yaabari Naenwi, Department of Electrical/Electronic Engineering, University of Cross River, Calabar, Cross River State, Nigeria.

compensator is to be designed using frequency response (Bode diagram design) approach, hopefully, this technique will foster control system engineer to design compensator precisely and conveniently with computer software called MATLAB /SIMULINK.

II. METHODOLOGY

The design of position control system of SEDCM include the following steps below and MATLAB/ Simulink is used for implementation:

1) Mathematical model for field control dc motor

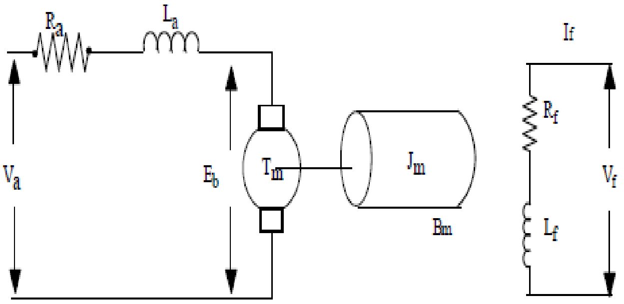

Fig 3.1 the equivalent circuit of separately excited DC motor [ref 2]

The field control of dc motor is adopted for position control as the rate of change of shaft angular position is inversely proportional to the flux per pole. When the flux is reduced, the speed of motor increased and also speed is directly proportional to armature voltage. In this scheme, armature current is kept constant.

The dynamic equations related to the speed control of dc motor as follows. Field flux (Φ) is proportional to field current (if)

The field winding flux (**Φ**) is proportional to the field current (I_f) while armature current is constant in Field Control scheme for Separately Excited DC Motor.

$$\Phi = K\Phi \,.\, If \qquad (1)$$

According to the Kirchhoff's voltage law (KVL) in armature circuit. The field winding voltage (Vf) of the DC motor can be expressed as:

$$V_f = L_f\left(\frac{di_f}{dt}\right) + R_f i_f \qquad (2)$$

Where Rf is winding resistance, L_f is field field winding inductance.

The developed torque on the motor shaft increases as the field current increases and vice versa, The relation between steady state or developed motor torque Tm, armature current (I_a), field current (I_f) and torque constant (K_t) is expressed as :

$$: T_m = K_t i_f i_a = K_t i_f \qquad (3)$$

Since armature current (ia) is constant .

From Newton's laws, the mechanical equation of motor can be described by the following equation,

$$T_m = J\left(\frac{d\omega}{dt}\right) + B\omega + T_L \qquad (4)$$

$$T_m = J\left(\frac{d^2\Theta}{dt^2}\right) + B\frac{d\Theta}{dt} + T_L \qquad (5)$$

Where J,Ө,$\boldsymbol{\omega}$, B are inertia of shaft, angular position of shaft, angular frequency and friction coefficient.

By applying Laplace transform to (2) and (4) with Zero (relaxed) initial condition, we have

$$\Theta(s) = \frac{K\Phi.\ If}{J.S^2 + B.s} \qquad (6)$$

$$V_f = [L_f.s + Rf].If \qquad (7)$$

The transfer function of **motor shaft position** with respect to input voltage is expressed as transfer function G(s) is expressed as

$$G(s) = \frac{\Theta(s)}{V_f(s)} = \frac{K\Phi}{s.[L_f.s + Rf].[Js + B]} \qquad (8)$$

The transfer function of **motor speed** with respect to input voltage is expressed as:

$$G(s) = \frac{\Omega(s)}{V_f(s)} = \frac{K\Phi}{[L_f.s + Rf].[Js + B]} \qquad (9)$$

The time constant transfer function of motor speed with respect to input voltage is expressed as K_m

$$\frac{\Omega(s)}{V_f(s)} = \frac{\left[\frac{K\Phi}{Rf.B}\right]}{\left[\frac{L_f}{Rf}.s+1\right].\left[\frac{J}{B}.s+1\right]} \qquad (10)$$

$$\frac{\Omega(s)}{V_f(s)} = \frac{Km}{[\tau f.s+1].[\tau m.s+1]} \qquad (11)$$

where I_f is field winding current, L_f is field winding Inductance , R_f is field winding Resistance, K_Φ is the motor flux constant, K_t is motor torque constant, R_a is armature winding Resistance, I_a is armature winding current which are kept constant, B is Viscous friction constant or frictional coefficient. (N.m/rad/sec), T_L is Load Torque (N.m), J is shaft moment of inertia (Kg.m2), ω and Ω are motor shaft speed (in rad/sec), Ө is shaft angular position, Km is motor gain constant, τ_f is field winding circuit time constant and τ_m is mechanical *time constant*.

Table 1 DC Motor data

Parameters	**Values**	**Units**
Field winding Resistance (R_f)	20	Ω
Field winding Inductance (L_f)	0.01	H
motor flux constant (K_Φ)	0.22	Wb / A
moment of inertia (J)	0.0044	Kg.m^2
Viscous friction constant (B)	0.012	N-m/(rad/sec)
Armature winding Resistance (R_a)	30	Ω
Armature winding Inductance (L_a)	0.017	H

By substituting motor data in table (1) in (8) for position control system using field flux control scheme we have

$$G(s) = \frac{\Theta(s)}{V_f(s)} = \frac{0.22}{s.[0.017.s+20].[0.0044s + 0.012]}$$

(12)

In term of motor time constant transfer function of position control system from (11) is derived to be

$$\frac{\theta(s)}{V_f(s)} = \frac{0.917}{s[0.0005.s+1].[0.367.s+1]}$$
(13)

Where km = 0.917, tm = 0.367 sec, tf = 0.0005 sec, and from (14). From (13) shows the position control system is third order-type 1 system, which are characterized with zero steady state error when step input voltage (V_f) is applied but exhibit steady state error when ramp input signal is applied.

2) Phase – Lead Compensator design

The phase lead compensators are designed to improve the transient response while lag compensators improve the steady state performance at expense of slower settling time. Therefore, lead compensator is design for position control in this section for fast transient response of the system.

The transfer function of phase lead compensator (controller) is given by

$$Gc(s) = \frac{Kc.[s+\frac{1}{\tau}]}{[s+\frac{1}{\alpha\tau}]} \qquad (14)$$

Where (0 < α < 1) and (t > 0)

The open loop transfer function (Go) of compensated system

$$G0 = Gc(s).KG(s) =$$

$$= \frac{Kc\ [s+\frac{1}{\tau}]}{[s+\frac{1}{\alpha.\tau}]} .\ KG(s) \qquad (15)$$

Where α = attenuation factor, t = Time constant

The phase-lead compensator is used to alter the response of position control system to accommodate the design specification or set criteria by introducing additional zero and poles to the system (plant). The plant in this case is the dc motor. The following are design specifications for the phase-lead compensated position–controlled system. Settling time (ts) < 2 seconds, Peak overshoot (p.0) < 4%, Phase margin (pm) > 66^0, Static velocity error constant (Kv) = 50 for unit ramp input.

3) Design Procedures for Lead Compensator using Frequency Response Techniques

(1). Determination of loop gain value (K = Kc α) to satisfy steady state error requirement (Kv)

From the transfer function of plant in (13) as G(s). The formula for static velocity error constant is given by

$$Kv = \lim_{s \to 0} s.KG(s) \qquad (16)$$

Therefore by substituting ( ) into (16) , we have Kv = 50 = 0.017**K**,

The required loop gain value for the system to meet the condition is that **K** = $K_c.\alpha$ = 54.54.

(2) The value of open loop transfer function for the compensator sated system is

$$KG(s) = \frac{\theta(s)}{V_f(s)} = \frac{[54.54]\,0.917}{0.0001835s^3+0.3675s^2+s} \quad (17)$$

(3) Bode diagram of uncompensated system in (16) using MATLAB software

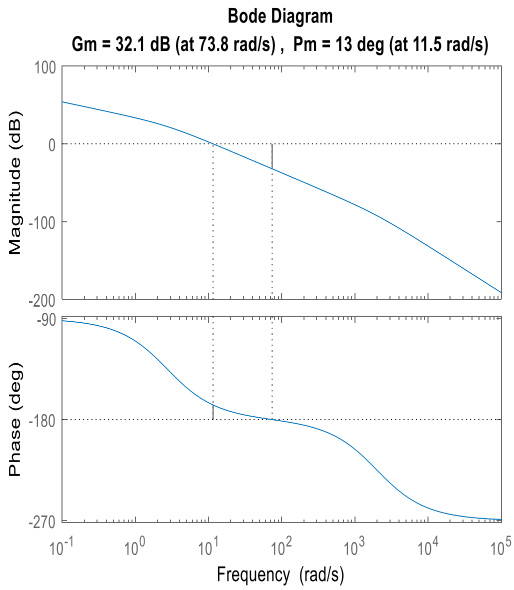

Fig 1 Bode diagram of uncompensated system G0(s)

from the figure1, the Gain Margin (Gm) is 32.1dB and Phase Margin (pm) = 13^0 and Since the design or desired pm is 66^0, we need a lead compensator to increase pm = 13^0

(4) Calculation of necessary phase lead angle

Phase lead angle (γm) is determined from expression given by

$$\gamma m = \gamma d - \gamma sys + \epsilon \qquad (18)$$

Where γd is designed phase margin, γsys is phase margin for open loop uncompensated system KG(s) and chosen correction factor (ϵ) range is (5^0 to 12^0) . From the design specification γm is ($66^0 - 13^0 + 5^0$) , $\gamma m = 58^0$. Since the phase lead angle $\gamma m < 60^0$ only one phase lead compensator is needed to increase the phase angle from 13^0 to target 66^0.

(5) Calculation of attenuation factor (α)

$$\alpha = \frac{1-\sin(\gamma m)}{1+\sin(\gamma m)} \qquad (19)$$

And α is 0.0824

(6). Calculation of new gain (An) at maximum lead frequency (ω_m)

$$An = -20\ Log\left[\frac{1}{\sqrt{\alpha}}\right] = 10\ Log\ [\alpha] \qquad (20)$$

By reading on the Bode diagram in figure 1, we can determine the maximum frequency (wm) by tracing at a point where the gain plot value is – 10.84dB. We have, $\omega_m = \omega_{gc}$ = 21.6 rad/s of Bode diagram [KGc(s)], ie new cross over frequency.

(7). Determination of time constant at maximum frequency (ω_m)

$$\tau = \frac{1}{\omega m.\sqrt{\alpha}} \qquad (21)$$

τ =0.161 Sec

(8). Determination of zero (ωz) and poles (ωp) (corner frequency)

$$\omega z = \frac{1}{\tau} \qquad (22)$$

$$\omega p = \frac{1}{\alpha.\tau} \qquad (23)$$

The corner frequencies are ωz = 6.1996 rad/s and ωp = 75.26 rad/s

(9). Determination of Lead compensator gain value (Kc)

$$Kc = \frac{1}{\alpha} \qquad (24)$$

Kc = 12.138

(10). Transfer function of phase lead compensator from (14) is obtained as

$$Gc(s) = \frac{12.14.[s+6.1996\ ]}{[S+75.26\ ]} \qquad (25)$$

(11) The open loop transfer function of the compensated system is Go(s) from (15)

$$G0(s) = \frac{[12.14.]\ [s+6.1996\ ]}{[s+75.26\ ]} \cdot \frac{[54.54].[0.917]}{[0.0001835s^3+0.3675s^2+s]}$$

(26)

Go(s)=

$$\frac{[607.16s+3764]}{[0.0001835s^4+0.3813s^3+28.66s^2+75.26s]}$$

(27)

(12) Bode diagram of the compensated position control system in (27)

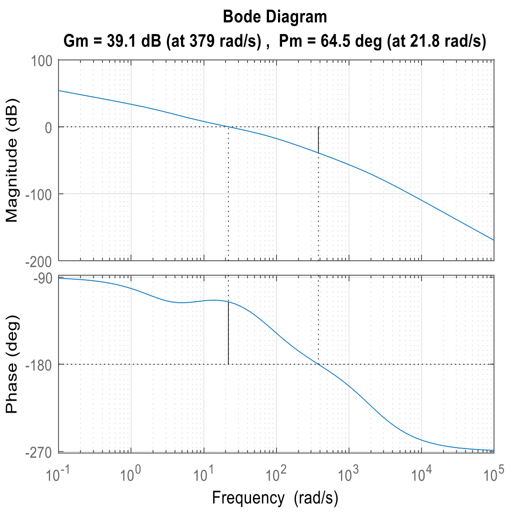

Fig 2 Bode diagram of compensated system Go(s)

We have phase margin (PM) of 64.5^0 at gain cross over frequency of 21. 8 rad/s, this is close to desired 66^0 and gain margin (GM) of 39. 1 dB at phase cross over frequency of 379 rad/s, this shows that the system is stable as Pm and GM are positive and PM of system has improve from 13^0 to 64.5^0 with phase Lead compensator.

With unity feedback H(s) = 1, The closed loop transfer function of compensated system is given by T (s)

T(s)=

$$\frac{[607.16s+3764]}{[0.0001835s^4+0.3813s^3+28.66s^2+682.4s+3764\,]}$$

(28)

4) Simulink model of closed loop compensated system

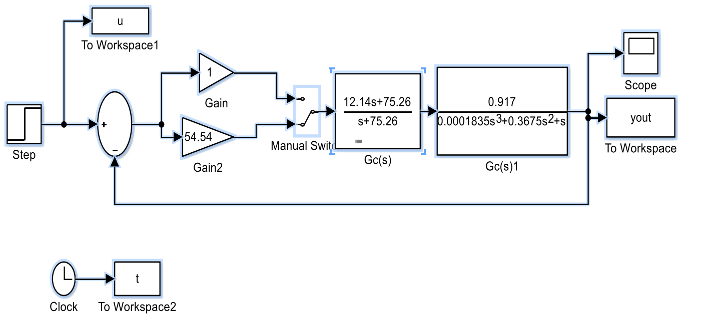

Fig 3 Simulink model of position control system

The unit step response and unity ramp response for closed loop compensated system is evaluated in Simulink diagram in figure 3 or using MATLAB command.

III. SIMULATION RESULTS AND DISCUSSION

The phase margin (pm) of compensated system is 64.5^0 closed to the design specification and the Gain Margin (GM) is 39.1dB which shows it is stable from figure 2.

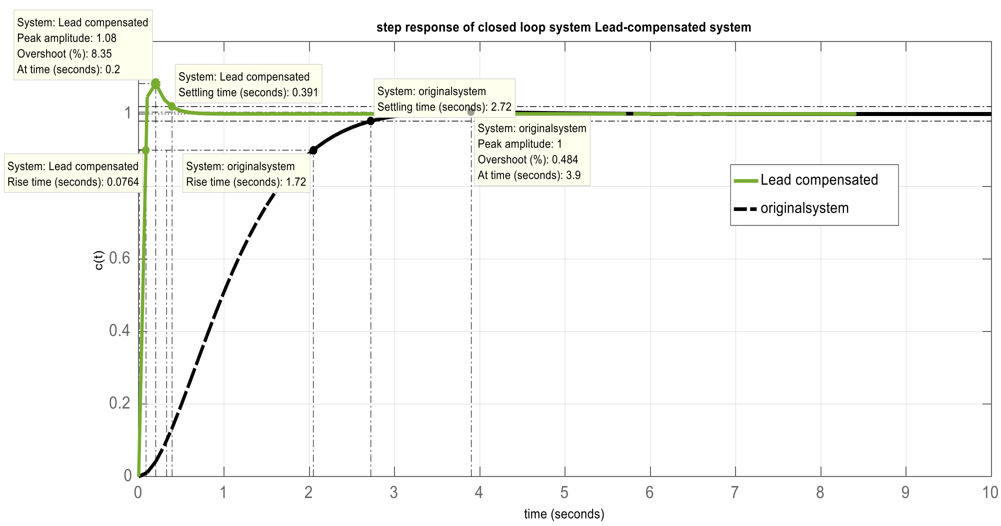

Fig 4 (a) step response of closed loop system with lead compensator correction factor $\epsilon = 5^0$

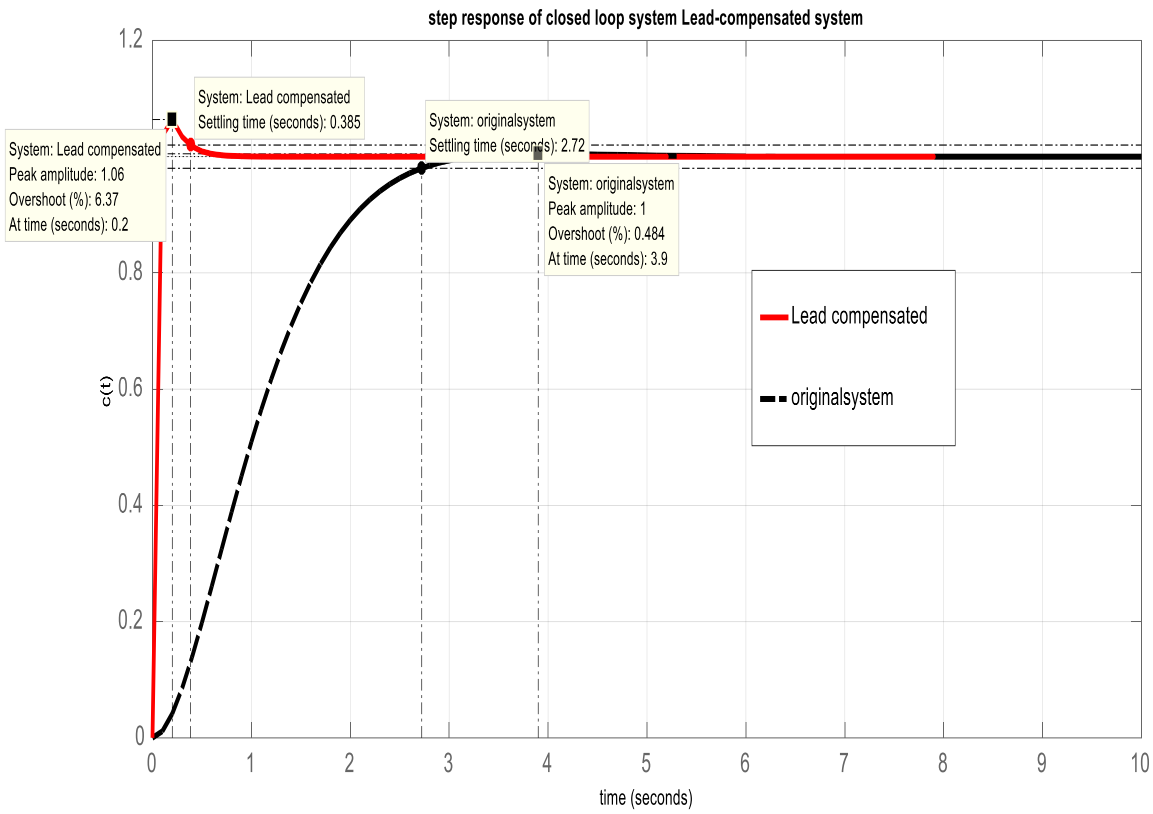

Fig 4 (s) step response with lead compensator with correction factor $\epsilon = 10^0$

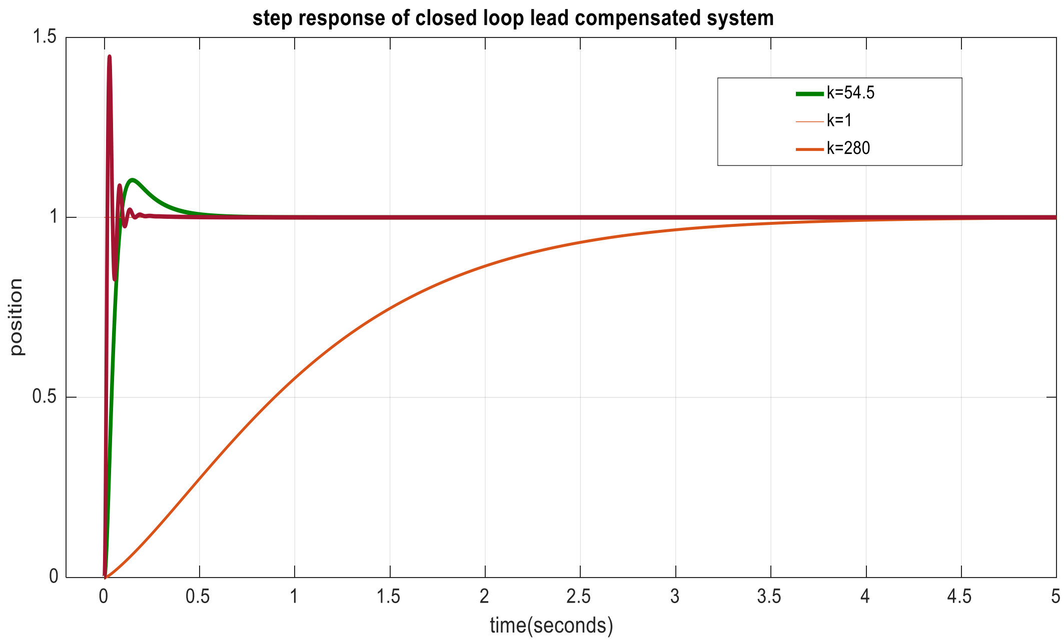

Fig 4 (b) step response with only loop gain closed loop system

This information on relative stability are positive values which are stable enough for the value of loop gain K in figure 2 is to be varied up to 39.dB ie, $K = 10^{(39/20)} = 89$ before the system becomes unstable ie, $K_{max} = (54.5 + 89 = 144)$. From figure 4, It is observed that if the K is varied beyond the gain margin (K> 144) the step response have oscillation and becomes worse when K > 280.

Table 1 : Performance indices of closed loop system with correction factor $\epsilon = 5^0$

Systems	**Setting time (ts) in seconds**	**Rise time (tr) in seconds**	**Peak Overshoot (P.O) in (%),**	**steady state error (ess)**
Uncompensated closed loop system	0.755	0.422	0	0.522
Compensated closed loop system	0.39	0.076	8.3	0

From the Speed response of closed loop system depicted in figure 4 have undesired value of 8.3 % that is greater than 4% given in table 1. when phase lead compensator is redesign with correction factor (ϵ) chosen to be = 10^0 we have performance improvement in table 2 as peak overshoot is less than 4 % desired value Where settling time is less than 2 seconds.

Table 2 **:** Performance indices of closed loop system with correction factor E = 10^0

Systems	**Setting time (ts) in seconds**	**Rise time (tr) in seconds**	**Peak Overshoot (P.O) in (%),**	**steady state error (ess)**
Uncompensated closed loop system	0.755	0.422	0	0.522
Compensated closed loop system	0.0981	0.08	0	(1-0.99) = 0.01

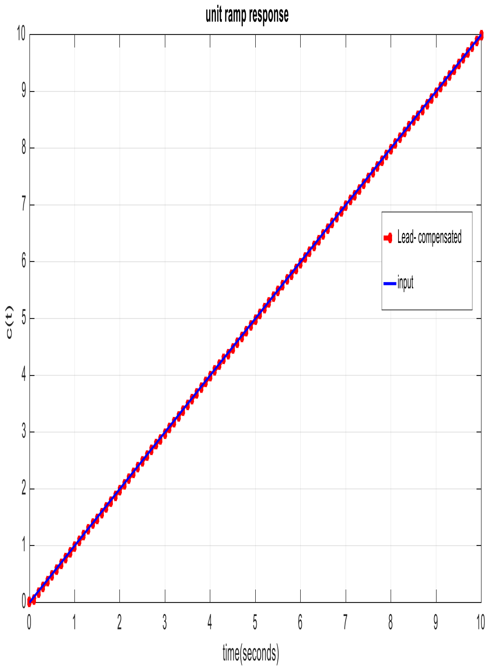

Fig 5(a) unit ramp response

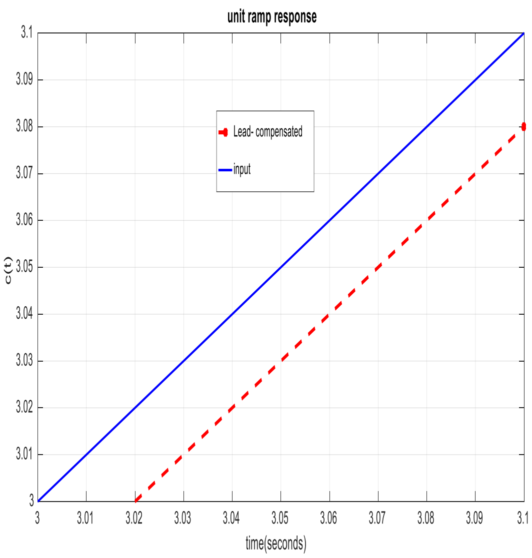

Fig 5(b) unit ramp response (Zoomed)

For the input of ramp signal to the closed loop system that is compensated system in figure 5b, we have the steady state error is **(3.03-3.01) = 0.02** and this gives **Kv** $= \frac{1}{0.02} =$ **50** which satisfy the design specification value at when **K = 54.5.**

CONCLUSION

Many position control and process control system utilize feedback control with controller or compensators, proper selection of compensator improves the system transient and steady state responses. In this paper, it is noticed that the designed compensator had improved the response of the SEDC motor. It satisfy the static error constant and phase margin requirement with faster settling time and peak overshoot slightly beyond the stated criteria due to very fast settling time that is below the specification. In overall, the design is satisfactory and easy to implement using MATLAB/Simulink software.

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