Barrier Function Based On Terminal Sliding Mode Control for Robotic Manipulators

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***Abstract*— This paper studies the optimal trajectory tracking control strategy of robotic manipulators in the presence of external disturbances. Super-twisting algorithm (STA) is used as switching controller to effectively weaken chattering. The choice of the barrier function as the gain of STA avoids the estimation of the disturbance upper bound and does not require the design of the low-pass filter. The stability of the closed-loop system is proved by Lyapunov theory. Finally, the effectiveness of the proposed method is verified by simulation experiments.**

***Index Terms*—Sliding mode control; Robotic manipulator; Barrier function; Super-twisting algorithm .**

I. Introduction

Nowadays, with the development of industries, robots are also being widely used in many fields to assist or replace humans in various tasks [1], such as aerospace [2], medical treatment [3], transportation [4], high precision welding [5] and so on. However, achieving fast and high-precision trajectory tracking for multi-joint robots has been a challenging goal due to the difficulty of obtaining accurate dynamic models and external disturbance that can significantly degrade the performance of the robot system. To achieve high-speed and high-precision trajectory tracking of robots, many control strategies have been proposed by scholars, including robust control [6], model predictive control [7], fuzzy control [8], and sliding mode control [9] (SMC).

SMC is always used in robot trajectory tracking research due to its advantages of fast response, good transient performance, easy implementation and tuning, and strong robustness against bounded external disturbances and system uncertainties [10]. In [11], an adaptive arctangent terminal SMC strategy is proposed, and a new adaptive law is designed to approximate the upper bound of the unknown disturbance. In [12], a robust adaptive fuzzy terminal SMC strategy with a low-pass filter is proposed for the trajectory tracking problem of a manipulator with external disturbance and dynamic uncertainty. This strategy can mitigate the adverse effects of model uncertainty and weaken the chattering of control inputs for stable control. In [13], an adaptive integral SMC strategy based on state observer is proposed for the trajectory tracking problem of flexible joint robots. A new adaptive law is also designed as the gain of the switching controller to eliminate the requirement for known disturbances and upper bounds on the uncertainty. However, all the above methods use adaptive strategies to estimate the upper bound of the external disturbance, and all of them need to solve the integral. This undoubtedly increases the amount of computation significantly.

The main objective of these techniques is to dynamically adjust the control gains in order to minimize them as much as possible while still providing sufficient counteraction against disturbances. These disturbances can be counteracted by increasing the gain to ensure the sliding mode is achieved. Once the sliding mode is reached, the high frequency control signal is filtered and used to provide information about the disturbance in the controller gain. The sliding mode controller gain is determined as the sum of the filtered signal and a constant value to compensate for any potential discrepancies between the real disturbance and its estimated value obtained through filtering. However, these requires knowledge of the minimum and maximum allowable values for the adaptive gain. According to these approaches, the gain increases until the sliding mode is achieved, and then decreases until the sliding mode is lost, indicating that the desired state is no longer being reached. These approaches ensure that the sliding variable converges to a neighborhood around zero within a finite time, without significantly overestimating the gain. The main limitation of these approaches is that the size of the aforementioned neighborhood and the convergence time depend on the unknown upper bound of disturbances, which cannot be known in advance.

Therefore, a novel adaptive SMC strategy based on barrier function is proposed for manipulator systems with model uncertainty and external disturbance. The barrier function is used as a gain strategy for STA, which does not require knowledge of the upper bound of the disturbance and weakens the chattering.

The rest of this paper is structured as follows: In Section II, the model description of the manipulator and some preparations are given. The barrier function-based STA terminal sliding mode controller is designed and its stability is analyzed in Section III. In Section IV, a set of simulation experiments demonstrate the effectiveness of the proposed method. Finally, the conclusion of this paper and the prospects for the future are given in Section V.

II. System modelling and problem statement

Consider the dynamics of a complete n-jointed robotic arm, described by the following second-order nonlinear differential equation:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)q=\tau+\tau_d \tag{1}$$

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where $M_q \in R^{2\times2}$ is the positive definite inertia matrix; $C(q,\dot{q}) \in R^{2\times2}$ is the vector of centrifugal force and coriolis force; $G(q) \in R^2$ is the vector of gravitational torques; $q \in R^2$ is the vectors of joint angular position, $\dot{q}$ and $\ddot{q}$ is the velocity and acceleration, $\tau \in R^2$ is the vector of joint control torque and $\tau_d \in R^2$ denotes the external disturbance. Here, we substitute $C(q,\dot{q})\dot{q}+G(q)$ with $L(q,\dot{q})$.

Alternatively, Eq. (1) can be written as:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}+\begin{bmatrix} -2b\dot{q}_2 & b\dot{q}_2 \\ b\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} +\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}=\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}+\begin{bmatrix} \tau_{d1} \\ \tau_{d2} \end{bmatrix} \tag{2}$$

where the q_1 and q_2 are the angular position of joints, τ_1 and τ_2 are the control torques. Other nonlinear functions are given:

$$\begin{aligned} a_{11} &= r_1^2(m_1+m_2)+2a_{12}-a_{22} \\ a_{12} &= a_{21} = a_{22}+r_1 r_2 m_2 \cos(q_2) \\ a_{22} &= r_2^2 m_2 \\ b &= r_1 r_2 m_2 \sin(q_2) \\ g_1 &= r_1(m_1+m_2)g\cos(q_1)+g_2 \\ g_2 &= r_2 m_2 g\cos(q_1+q_2) \end{aligned} \tag{3}$$

where r_1 and r_2 are the Length of lever, m_1 and m_2 are the mass, g is an vector of gravitational terms.

Considering the system uncertainty, the actual dynamics matrix and vector are described as follows:

$$\begin{aligned} M(q) &= M_0(q)+\Delta M \\ C(q,\dot{q}) &= C_0(q,\dot{q})+\Delta C \\ G(q) &= G_0(q)+\Delta G \\ F(q) &= F_0(\dot{q})+\Delta F \end{aligned} \tag{4}$$

where ΔM, ΔC and ΔG are system model uncertainties, G_0 is thethe vector of gravitational torques, M_0 is the positive definite inertia matrix and $C(q,\dot{q}) \in R^n$ is the vector of centrifugal force and coriolis force, and $x^T(M_0(q)-2C_0(q,\dot{q}))x=0$, which $M_0(q)-2C_0(q,\dot{q})$ is a skew symmetric matrix.

Assumption: *$\|M_0\|$ is bounded, and $\mu_1 \le \|M_0\| \le \mu_2$, which μ_1 and μ_2 are known normal number.*

Assumption: *C_0 and G_0 satisfy the equations $\|C_0\| \le \bar{c}$ and $\|G_0\| \le \bar{g}$, which $\bar{c}$ and $\bar{g}$ are normal numbers.*

Substituting Eq. (4) into Eq. (1), we can get:

$$M_0(q)\ddot{q}+C_0(q,\dot{q})\dot{q}+G_0(q)=\tau+\tau_d+I(q,\dot{q},\ddot{q}) \tag{5}$$

where $I(q,\dot{q},\ddot{q})$ is the unknown uncertainty and $I(q,\dot{q},\ddot{q})=\Delta M(q)\ddot{q}+\Delta C(q,\dot{q})\dot{q}+\Delta G(q)$, which is bounded and the upper bound has already been given in [14].

The following unequal are give:

$$\|I(q,\dot{q},\ddot{q})\| \le c_0+c_1\|q\|+c_2\|\dot{q}\|^2 \tag{6}$$

where c_0, c_1 and c_2 are positive constants. To reduce system disturbances and enhance robustness, the uncertainty and external disturbances τ_d is regarded as the disturbance of system $H(q,\dot{q},\ddot{q},t)$. One can get:

$$M_0(q)\ddot{q}+L_0(q,\dot{q})=\tau+H(q,\dot{q},\ddot{q},t) \tag{7}$$

Assumption:*The disturbances of the closed-loop robotic manipulator arm system is bounded and complies with the $\|H(q,\dot{q},\ddot{q},t)\| \le d_0$, $\|\dot{H}(q,\dot{q},\ddot{q},t)\| \le d_1$, which d_0 and d_1 are bounded constants.*

Definition:*According to the [15] and [16], in this study, the barrier function is given as follows:*

$$L_b(x)=\frac{\varepsilon b}{\varepsilon-|x|}, x \in (-\varepsilon,\varepsilon) \tag{8}$$

where the b and ε are positive constant.

Considering the implementation of the barrier function, we define the $L(t,s)$ as:

$$L(t,s)=\begin{cases} \rho t+\zeta, 0 \le t < t_1 \\ L_b(s), t \ge t_1 \end{cases} \tag{9}$$

For the definition of this barrier function, L_0 and L_1 are arbitrary positive constants.

Lemma:*[17] For this system:*

$$\begin{cases} \dot{x}_1 = -k_1|x_1|^{1/2}\operatorname{sign}(x_1) \\ \dot{x}_2 = -k_2\operatorname{sign}(x_1) \end{cases} \tag{10}$$

considering $\zeta^T=[\zeta_1,\zeta_2]=[|x_1|^{1/2}\operatorname{sign}(x_1), x_2]$, one can get:

$$\dot{\zeta}=\frac{1}{|\zeta_1|}A\zeta \tag{11}$$

$$A=\begin{bmatrix} -\frac{1}{2}k_1 & \frac{1}{2} \\ -k_2 & 0 \end{bmatrix} \tag{12}$$

If there exist $V(x)=\zeta^T P\zeta$ and

$\dot{V}(x) = -|x_1|^{1/2} \zeta^T Q \zeta$ *and satisfy this equation:*

$$A^T P + PA = -Q \tag{13}$$

which P, Q *are positive definite matrices and they are both symmetric. Based on these descriptions, and considering* T_0 *as the initial time, the systems (10) will converge to the origin in finite time* T_1.

$$T_1 = \frac{2}{\sigma} V^{1/2}(T_0) \tag{14}$$

where $\sigma = \dfrac{\lambda_{\min}^{1/2}\{P\}\lambda_{\min}\{Q\}}{\lambda_{\max}\{P\}}$.

III. Design of controller

For the system (1), $q \in R^2$ is the vectors of joint angular position, $q_d \in R^2$ is used to represent the position vector of the desired joint angle.

Define the tracking error as follows:

$$e = q - q_d \tag{15}$$

The time derivative of the error can be shown as:

$$\dot{e} = \dot{q} - \dot{q}_d \tag{16}$$

Considering the property of finite time convergence, the designed terminal sliding mode is expressed as:

$$s = \dot{e} + \beta e^{\alpha} \tag{17}$$

where $\beta = [\beta_1, \beta_2]^T$ and $\alpha = [\alpha_1, \alpha_2]^T$ are positive parameter matrices.

The equivalent controller is designed as follows:

$$\tau_{eq} = M^{-1}(C(q,\dot{q})\dot{q} + G(q)) - \beta\alpha e^{\alpha-1} + \ddot{q}_d \tag{18}$$

The switching controller is designed as follows:

$$\tau_{sw} = -K_1 |s|^{\frac{1}{2}} sign(s) - \int_0^t K_2^{\;2} sign(s) d\tau \tag{19}$$

The $K_1 \in R^{2\times1}$ and $K_2 \in R^{2\times1}$ in (\ref{eq19}) are control gains, the implementation of τ_{sw} requires the unknown upper bound of disturbance, and the value of K_1 and K_2. Consider the following variable gain of the switching controller that solves this problem.

$$\tau_{sw} = -L_1(t,s)|s|^{\frac{1}{2}} sign(s) - \int_0^t L_2(t,s)^{2} sign(s) d\tau \tag{20}$$

where the $L(t,s)$ is the variable gain.

The proposed controller can be represented as follows:

$$\tau = \tau_{sw} + \tau_{eq} \tag{21}$$

Theorem: *Under the action of the control signal (21), the obotic manipulator tracks the desired trajectory in finite time.*

Proof: *Setting the following variable:*

$$z = [z_1, z_2]^T = \left[|s|^{\frac{1}{2}} sign(s), \int L_2^{\,2} sign(s) dt\right]^T \tag{22}$$

And we can take the derivative of (22).

$$\dot{z} = \frac{1}{2|z_1|} Az, A = \begin{bmatrix} -\frac{1}{2}L_2 & 1 \\ -L_2^{\,2} & 0 \end{bmatrix} \tag{23}$$

Consider the Lyapunov function:

$$V_1 = z^T P z \tag{24}$$

where P is a symmetric positive-definite matrix:

$$P = \begin{bmatrix} b & -3d \\ -3d & 1 \end{bmatrix} \tag{25}$$

where a and b are position numbers and $b > 9d^2$. Taking the derivative of V_1 as follows:

$$\dot{V}_1 = z^T\left[A^T P + PA\right]z \le -\frac{1}{2|z|} z^T Q z \tag{26}$$

where $-Q = A^T P + PA$. According to the definition of barrier function in (9), we divide the proof into two parts:

1) When $0 \le t \le t_1$: We define the matrices A and Q separately:

$$A = \begin{bmatrix} -(L_1 t + L_0) & 1 \\ -(L_1 t + L_0)^2 & 0 \end{bmatrix}$$
$$Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_3 & Q_4 \end{bmatrix} \tag{27}$$

where $Q_1 = -2b(L_1 t + L_0) - 3d(L_1 t + L_0)^2$ $Q_2 = Q_3 = a + 3d(L_1 t + L_0) - (L_1 t + L_0)^2$, and $Q\_4 = 0$. According to lemma, $T_n = \frac{2}{\sigma} V_1^{\frac{1}{2}}(T_0)$ and we can know sliding surface s will converge to $(-\varepsilon, \varepsilon)$ in finite time:

$$T_1 = T_0 + T_n \tag{28}$$

2) When $t \ge t_1$: We define the matrices A and Q separately:

$$A = \begin{bmatrix} -\frac{1}{2}\frac{\varepsilon b}{\varepsilon - |s|} & 1 \\ -\frac{\varepsilon^2 b^2}{(\varepsilon - |s|)^2} & 0 \end{bmatrix}$$
$$Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_3 & Q_4 \end{bmatrix} \tag{29}$$

$$T_s = T_1 + T_m \tag{30}$$

where $Q_1 = -2\frac{\varepsilon b}{\varepsilon - |s|}(b + 9d^2) - 2\frac{\varepsilon^2 b^2}{(\varepsilon - |s|)^2}$, $Q_2 = Q_3 = b + 9d^2 + 3d\frac{\varepsilon b}{\varepsilon - |s|} - \frac{\varepsilon^2 b^2}{(\varepsilon - |s|)^2}$, and $Q_4 = 0$.

According to lemma, $T_m = \frac{2}{\sigma}V_1^{\frac{1}{2}}(T_1)$, we can know s will converge to zero in finite time:

IV. Simulation Experiment

This section describes the parameter selection as well as the experimental analysis.

The barrier funtion: Slect $t_1 = 0.5s$,when $0 < t < t_1$: for $L_1(t, s)$: L_1=7000 and L_0=1 ; for $L_2(t, s)$: L_1=500 and L_0=5 .When $t > t_1$:for $L_1(t, s)$: ε=0.1 and b=600 ; for $L_2(t, s)$: ε=0.1 and b=100 . And when $0 < t < t_1$: for $L_1(t, s)$: L_1=1 and L_0=1 ; $L_2(t, s)$: L_1=1 and L_0=1 , when $t > t_1$: for $L_1(t, s)$: ε=0.1 and b=50 ; for $L_2(t, s)$: ε=0.1 and b=50 .

For the equivalent controller, we choose the following parameters: the desired trajectory is designed as $q_d = [\sin(\pi t), \cos(\pi t)]^T$. However, for q_1 and q_2, we use a different expression for the sliding mode. $\beta = [17, 20]^T$, $\alpha = [0.9, 0.9]^T$.

For the system, we choose the following parameters: $m_1 = 10$, $m_2 = 1$, $r_1 = 1$, $r_2 = 1$ and $g = 9.8$ in (3). And the external disturbance $\tau_d = [\sin(\frac{20}{\pi t}), \cos(\frac{20}{\pi t})]^T$.

The controller incorporating the barrier function which is described in the previous section demonstrates that the proposed scheme can guarantee the robotic manipulator to track the desired trajectory in finite time. In order to demonstrate the reliability of the proposed scheme, the detailed analysis of the simulation results is presented as follows:

Fig. 1 depict the evolution of system states q_1, q_2 and the desired trajectory. According to the Fig. 1, it can be observed that q_1 can track the desired trajectory within 0.12s, and q_2 can approach the desired signal within 0.21s. It's worth noting that both q_1 and q_2 tend to stable and there is no divergence trend in the simulation period.

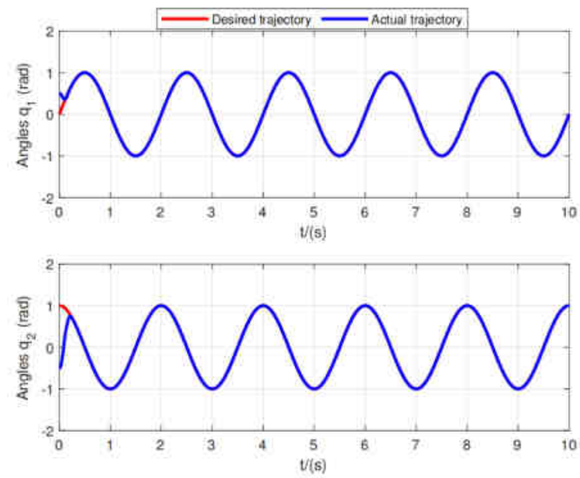

Fig 1. The tracking results of q_1 and q_2 .

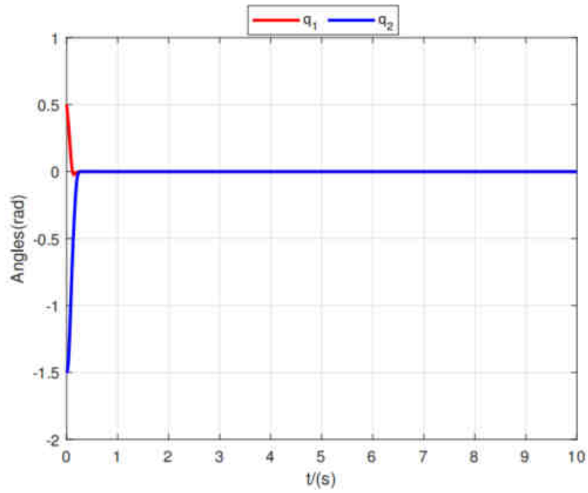

Fig 2. The tracking errors of q_1 and q_2 .

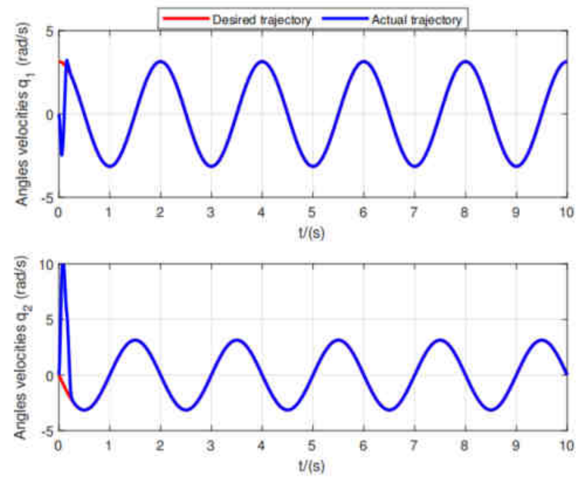

Fig 3. The derivative of tracking results.

Fig. 2 shows the tracking errors of angles q_1 and q_2 , respectively. Moreover, the stability of tracking errors can be ensured in finite time, indicating that q_1 and q_2 have successfully tracked the desired trajectories.

Fig. 3 illustrates the derivative of the system states and the desired trajectories about $\dot{q}_{1d}$ and $\dot{q}_{2d}$. The correctness of the derivative of the desired trajectory in Fig. 3 can be observed. Similarly to Fig. 1, the differentiated value of the actual trajectory is also consistent with the differentiated value of the desired trajectory over the same period of time.

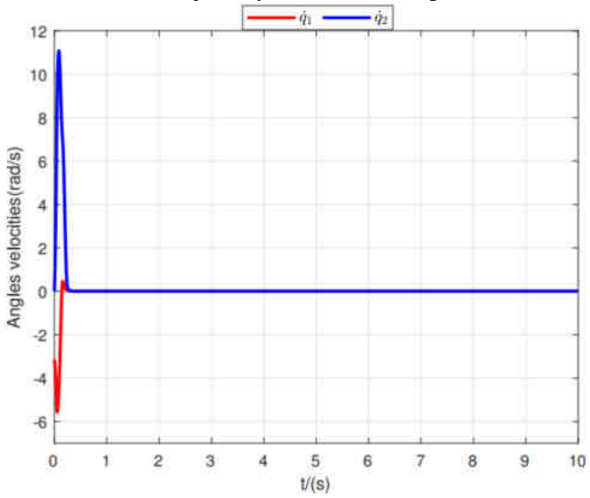

Fig 4. The derivative of tracking errors.

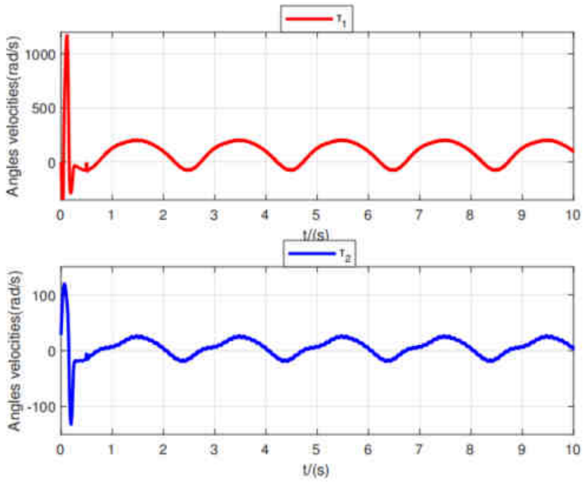

Fig 5. Control input τ_1 and τ_2.

Fig. 4 illustrates the derivative of the tracking error. It can be seen that the derivative of the error of q_1 shows an increase and then a decrease, before converge to 0. The derivative of the error of q_2 decrease firstly and then converge to 0.

In addition, the control inputs of τ_1 and τ_2 are presented in Fig. 5 demonstrating that the control action is really smooth, which means the chattering is mitigated and be used in real applications. Fig. 6 illustrates the evolution of the sliding surface s_1 and s_2. s_1 and s_2 converge to 0 within 0.12s and 0.21s, respectively. What's more, the chattering of sliding surface is mitigated according to the proposed adaptive super-twisting algorithm.

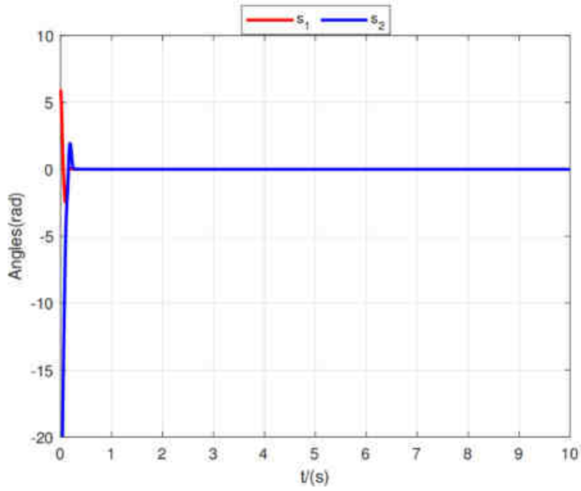

Fig 4. Sliding surfaces s_1 and s_2.

V. CONCLUSION

In this paper, we propose a novel barrier function based STA SMC strategy for trajectory tracking of manipulators. Combining the barrier function with the STA, this strategy does not require knowledge of the upper bound of the disturbance, nor does it require a low-pass filter, and is capable of achieving finite-time stability of the system while weakening the chattering. Finally, the simulation results verify the effectiveness of the proposed method.

Future work is the application of the proposed method in a practical industrial manipulator.

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