Observer-Based Event-Triggered Sliding Mode Control for Attitude of Quadrotor with Mismatched Disturbances

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***Abstract*— This paper presents an observer-based event-triggered adaptive super-twisting controller for attitude tracking of a quadrotor unmanned aerial vehicles (UAVs) in the presence of matched and mismatched disturbances. Firstly, an observer-based adaptive super-twisting controller is designed to mitigate overestimation of gains and reduce chattering effects, thereby effectively addressing both matched and mismatched disturbances, and enhancing the control performance and robustness of quadrotor UAV. Furthermore, a dynamic event-triggered control strategy is employed, which flexibly determines the timing of controller updates based on the actual state variations of the system. This approach reduces unnecessary controller updates, saving communication and computational resources, and enhances system performance and efficiency. Finally, the superiority of the proposed method is validated through simulation and experimental results.**

***Index Terms*—Quadrotor UAV, Super-twisting algorithm, Adaptive sliding mode control, Disturbance observer, Event-triggered control**

I. Introduction

Quadrotors UAVs are complex nonlinear systems with weak disturbance rejection capabilities. During actual flight, they are affected by both matched and mismatched disturbances. Matched disturbances such as air resistance and gravity can be compensated for by adjusting the control system. However, handling mismatched disturbances such as model uncertainties and sensor errors is more challenging.

To overcome these challenges, researchers have explored a range of linear control strategies for achieving precise control of quadrotors [1]-[3]. However, as control requirements continue to increase, the limitations of linear control strategies become more apparent. They are sensitive to parameter variations in nonlinear systems. As a result, various nonlinear control methods have emerged and are widely applied in quadrotors control [4]-[6]. Sliding mode control (SMC), known for its robustness and insensitivity to nonlinear uncertainties and external disturbances, has gained more popularity in quadrotors applications. However, SMC suffers from chattering due to rapid switching caused by the sliding mode switching structure. To overcome this issue, the super -twisting algorithm has been proposed and successfully applied in robust control.The super-twisting controller relies on knowledge of the disturbance upper bound, which is often difficult to accurately determine in practical situations. Some researchers have addressed this problem by designing methods with adaptive gains, effectively reducing the chattering [7]-[9]. Furthermore, the presence of model uncertainties and sensor errors in quadrotors can also lead to mismatched disturbances in the system, which cannot be completely suppressed by the SMC thus impacting control performance and stability. To address this problem, disturbance observers (DO) have been introduced to estimate and compensate for the mismatched disturbances, as they can directly compensate for the uncertainties in the system without affecting the performance of the original control method. Research scholars have focused on incorporating DO to enhance control performance and stability [10]-[11].

In addition, for resource-limited quadrotors, traditional fixed-period sampling control may not meet the requirements of high workload operations. To conserve communication and computational resources, some literature has proposed event-triggered control strategies [12]-[13]. However, these studies have shown a dependence of the system state on the triggering parameters, which limits the possibility of infinitely increasing the event interval time. Moreover, to ensure system stability, the control switching gain noticeably relies on the triggering parameters. In order to address these limitations, several studies have proposed improvements to control algorithms for sparse trigger sequences by utilizing adaptive trigger parameters and designing dynamic trigger parameters [14]-[15]. These studies have proposed effective solutions to better conserve resources considering the presence of matched disturbances. However, they are not effective in dealing with the presence of mismatched disturbances.

In this paper, inspired by the aforementioned investigations, a novel event-triggered adaptive super-twisting control method based on an observer is proposed. By dynamically adjusting the gain, the overestimation of the gain is avoided, reducing the chattering phenomenon. Additionally, by combining the observer-based event-triggered mechanism with adaptive super-twisting method, the issue of mismatched disturbances is effectively addressed while conserving energy

II. Quadrotor attitude model

The model of quadrotor attitude can be given[16]-[17]

$$\ddot{\Theta}=[J\psi(\Theta)]^{-1}[U-N(\Theta,\dot{\Theta}+d_1)(\dot{\Theta}+d_1)]+d_2 \tag{2.1}$$

The state vectors are defined as:

$$x_1=\Theta, x_2=\dot{\Theta} \tag{2.2}$$

The dynamic model is reformulated using the state-space format as follows:

$$\begin{cases}\dot{x}_1=x_2+d_1\\ \dot{x}_2=f(x,t)+g(x,t)U+d_2\end{cases} \tag{2.3}$$

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where $f(x,t) = -[J\psi(x_1)]^{-1} N(x_1;\dot{x}_1)\dot{x}_1$, $g = [J\psi(x_1)]^{-1}$, d_1 represents mismatched disturbances from a different channel than the control input, d_2 represents matched disturbance from the same channel as the control input.

Before commencing the controller design, we provide the following assumptions and definitions:

Assumption 1: Disturbances $d_i(x,t)$ and their time derivatives in the quadcopter unmanned aerial vehicle system are bounded and satisfy $\|\dot{d}_i\| \le \delta$.

Assumption 2: The function F' satisfies the Lipschitz property, meaning for a Lipschitz constant L and for all $x_1, x_2 \in R^{6\times1}$ within a closed set D is valid as

$$\|F'(x_1) - F'(x_2)\| \le L\|e(t)\| \tag{2.4}$$

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III. ATTITUDE CONTROLLER DESIGN

To mitigate the influence of disturbances, the finite-time observer can be employed to approximate the disturbances:

$$\begin{aligned}
&\dot{z}_0^i = v_0^i + f_i(x,u), \dot{z}_1^i = v_1^i, \dots, \dot{z}_{n-i+1}^i = v_{n-i-1}^i \\
&v_0^i = -\lambda_0^i L_i^{\frac{1}{n-i+2}} \left|z_0^i - x_i\right|^{\frac{n-i+1}{n-i+2}} sign(z_0^i - x_i) + z_1^i, \\
&v_j^i = -\lambda_j^i L_i^{\frac{1}{n-i+2-j}} \left|z_j^i - v_{j-1}^i\right|^{\frac{n-i+1-j}{n-i+2-j}} sign(z_j^i - v_{j-1}^i) + z_{j+1}^i, \\
&v_{n-i+1}^i = -\lambda_{n-i+1}^i L_i sign(z_{n-i+1}^i - v_{n-i}^i), \\
&\hat{x}_i = z_0^i, \hat{d}_i = z_1^i, \hat{\dot{d}}_i = z_2^i, \dots, \hat{d}_i^{[n-i]} = z_{n-i+1}^i,
\end{aligned} \tag{3.1}$$

where $i = 1,2,\dots,n$, $j = 1,2,\dots,n-i+1$, $f_i(x,u) = f + gu$, $\lambda_j^i > 0$ is the observer coefficient, $\hat{x}_i, \hat{d}_i, \hat{\dot{d}}_i, \hat{d}_i^{[n-i]}$ are the estimated value of the corresponding actual variable $x_i, d_i, \dot{d}_i, d_i^{[n-i]}$.

The quadrotor attitude tracking error is defined as follows: $\tilde{x} = [\tilde{x}_1^T \quad \tilde{x}_2^T]^T \in R^{6\times1}$, $\tilde{x}_1 = x_1 - x_{1d} = [e_\phi, e_\theta, e_\psi]^T$, $\tilde{x}_2 = \dot{x}_1 - \dot{x}_{1d}$, where $x_{1d} \in R^{3\times1}$ is the desired attitude vector.

Firstly, design the sliding mode as: $s = C\tilde{x}' = \Lambda\tilde{x}_1 + \tilde{x}_2 + e_1^1 \in R^{3\times1}$, $C = [\Lambda \quad I] \in R^{3\times6}$. $\Lambda_{3\times3}$ is a diagonal matrix, then the sliding surface is defined as:

$$S := \left\{\tilde{x} \in R^{6\times1} : s = C\tilde{x}' = \Lambda\tilde{x}_1 + \tilde{x}_2 + e_1^1 = 0\right\} \tag{3.2}$$

where, $e_1^1 = \hat{d}_1 - d_1$, we can set $\tilde{x}' = [\tilde{x}_1^T \quad \tilde{x}_2'^T]^T \in R^{6\times1}$, where $\tilde{x}_2' = x_2 + \hat{d}_1 - \dot{x}_{1d}$, we can get:

$$\dot{\tilde{x}}' = \begin{bmatrix} x_2 - \dot{x}_{1d} \\ F \end{bmatrix} + BU' + \begin{bmatrix} d_1 \\ d_2 + \dot{\hat{d}}_1 \end{bmatrix} = F' + BU' + D' \tag{3.3}$$

where $\tilde{x}' = [\tilde{x}_1; \tilde{x}_2'] \in R^{6\times1}$, $F' = [x_2 - \dot{x}_{1d}; F] \in R^{6\times1}$, $B = [0^{3\times3}; I^{3\times3}] \in R^{6\times3}$, $U' \in R^{3\times1}$, $D' = [d_1; d_2 + \dot{\hat{d}}_1]$.

The sliding mode attitude controller is designed as follows:

$$\begin{aligned}
U'(t) = (CB)^{-1}(&-CF'(t_i) - C\hat{D} - K_1\|s(t_i)\|^{\frac{1}{2}} sign(s(t_i)) - \\
&K_2\int_{t_i}^{t} sign(s(t_i))dt
\end{aligned} \tag{3.4}$$

where, $\hat{D} = [\hat{d}_1; d_2 + \dot{\hat{d}}_1] \in R^{6\times1}$, meanwhile, the adaptive gains are designed as:

$$\dot{K}_1(t) = \begin{cases} \omega_1\sqrt{\dfrac{1}{2}}\, sign(|s(t)| - \mu), K_1(t) > K_m \\ \eta, K_1(t) \le K_m \end{cases} \tag{3.5}$$

and $K_2 = \text{ò}K_1$.

Define the sampling error as: $e(t) = \tilde{x}'(t_i) - \tilde{x}'(t)$, $t \in [t_i, t_{i+1})$.

Firstly, introduce the scalar dynamics in the triggering rule as:

$$\dot{\lambda}(t) = -\beta(\lambda(t)) - \lambda(t)s(t) + (|s(t)|^{\frac{1}{2}} - L\|C\|\|e(t)\|)|s(t)| \tag{3.6}$$

where, let $\beta(\cdot)$ be a locally Lipschitz continuous function, define $\Lambda_1 = \{\lambda \in R : 0 \le \lambda \le \lambda_M\}$, and $\lambda_M > 0$, $\Lambda_2 = \{\lambda \in R : 0 \le \lambda \le |s|^{\frac{1}{2}}\}$.

The dynamic event-triggering rule is defined as follows:

$$t_{i+1} = \inf\{t > t_i : L\|C\|\|e(t)\| \ge \sigma\|s(t_i)\|^{\frac{1}{2}} + \lambda(t)F_{\Lambda_2}(\lambda(t))\} \tag{3.7}$$

where L is a Lipschitz constant, $\sigma \in (0,1)$, $F_{\Lambda_2}(\lambda)$ is defined as:

$$F_{\Lambda_2}(\lambda) = \begin{cases} 1, & if \ \lambda \notin \Lambda_2 \\ 0, & if \ \lambda \in \Lambda_2 \end{cases} \tag{3.8}$$

This triggering condition ensures that for $t \ge 0$:

$$L\|C\|\|e(t)\| < \sigma\|s(t_i)\|^{\frac{1}{2}} + \lambda(t)F_{\Lambda_2}(\lambda(t)) \tag{3.9}$$

The following theorem provides the stability proof for the proposed scheme.

Theorem 1: Consider the system (3.3) under the conditions of Assumption 1 and Assumption 2. Applying the controller composed of (3.4) and (3.5), along with the triggering rules (3.6) and (3.7), if the control gains satisfy:

$$K_1 > \frac{\theta + 4\text{ò}^2 + \text{ò}}{\theta} + \frac{1}{4\text{ò}\theta}(\theta + 4\text{ò}^2 + 2\text{ò})^2 \tag{3.10}$$

Then the sliding surface will converge to the following domain:

$$\Omega = \left\{\|s(t)\| \le \max\{\frac{M^2}{\lambda_{\min}^2(Q)\vartheta^2}, \frac{\sigma\|s(t_i)\|^{\frac{1}{2}} + \lambda_M}{L}\}\right\} \tag{3.11}$$

Proof: By considering (3.3) and (3.4), differentiation of the sliding variable (3.2) yields:

$$\begin{aligned}
\dot{s} = &CF'(t) - CF'(t_i) - K_1\|s(t_i)\|^{\frac{1}{2}} sign(s(t_i)) \\
&- K_2\int_{t_i}^{t} sign(s(t_i))dt - C\tilde{D}
\end{aligned} \tag{3.12}$$

where $\tilde{D} = [\hat{D} - D] = [e_1^1; e_1^2] \in R^{6\times1}$.

Introducing $e_0^i = z_0^i - x_i$, $e_j^i = z_j^i - d_i^{[j-1]}$, e_0^i represent the observation errors of the system states x_i, and e_1^i to represent the observation error of the non-matching disturbance d_i, we can obtain the estimation error of the observer as:

$$\begin{aligned}
&\dot{e}_0^i = -\lambda_0^i L_i^{\frac{1}{n-i+2}} \left|e_0^i\right|^{\frac{n-i+1}{n-i+2}} \operatorname{sgn}\left(e_0^i\right) + e_1^i, \\
&\dot{e}_j^i = -\lambda_j^i L_i^{\frac{1}{n-i+2-j}} \left|e_j^i - \dot{e}_{j-1}^i\right|^{\frac{n-i+1-j}{n-i+2-j}} \operatorname{sgn}\left(e_j^i - \dot{e}_{j-1}^i\right) + e_{j+1}^i, \\
&\dot{e}_{n-i+1}^i \in -\lambda_{n-i+1}^i L_i \operatorname{sgn}\left(e_{n-i+1}^i - \dot{e}_{n-i}^i\right) + [-L_i, L_i],
\end{aligned} \tag{3.13}$$

It follows from [18] that the system (3.13) is finite-time stable. Regardless of the system state x_i, the observer estimation error e_i^j will converge to zero within a finite time.

Therefore, the sliding variable (3.12) can be simplified to:

$$\dot{s} = CF'(t) - CF'(t_i) - K_1\|s(t_i)\|^{\frac{1}{2}} sign(s(t_i)) - K_2\int_{t_i}^{t} sign(s(t_i))dt \tag{3.14}$$

Step 1: Prove the convergence of the sliding variable to the domain (3.11). Let $\omega(t) = -K_2\int_{t_i}^{t} sign(s(t_i))dt$, equation (3.14) becomes:

$$\begin{cases} \dot{s}(t) = CF'(t) - CF'(t_i) - K_1\|s(t_i)\|^{\frac{1}{2}} sign(s(t_i)) + \omega(t) \\ \dot{\omega}(t) = -K_2 sign(s(t_i)) \end{cases} \tag{3.15}$$

Define $\xi(t)=[\xi_1(t),\xi_2(t)]^T$, where $\xi_1(t)=|s(t)|^{\frac{1}{2}}sign(s(t))$, $\xi_2(t)=\omega(t)$. Taking the derivative with respect to $\xi(t)$, we can get:

$$\dot{\xi}(t)=\frac{1}{\|\xi_1(t)\|}\left\{\underbrace{\begin{bmatrix}-\frac{K_1}{2} & \frac{1}{2}\\ 0 & 0\end{bmatrix}}_{A_1}\begin{bmatrix}\xi_1(t_i)\\ \xi_2(t)\end{bmatrix}+\underbrace{\begin{bmatrix}\frac{1}{2}C[F'(t)-F'(t_i)]\\ -K_2\|\xi_1(t)\|sign(s(t_i))\end{bmatrix}}_{A_2}\right\} \quad (3.16)$$

$$=\frac{1}{\|\xi_1(t)\|}(A_1\begin{bmatrix}\xi_1(t_i)\\ \xi_2(t)\end{bmatrix}+A_2)$$

Introduce the following Lyapunov function for stability analysis of the system:

$$V_{(\xi,K)}=V_{(\xi)}+\frac{1}{2}(K_1-K_1^*)^2+\frac{1}{2}(K_2-K_2^*)^2 \quad (3.17)$$

where:

$$V_{(\xi)}=\xi^T(t)P\xi(t) \quad (3.18)$$

The positive definite matrix P is defined as:

$$P=\begin{bmatrix}\theta+4\grave{o}^2 & -2\grave{o}\\ -2\grave{o} & 1\end{bmatrix} \quad (3.19)$$

Due to the switching structure of the designed triggering mechanism, two cases are analyzed for the convergence of the closed-loop system trajectories to (3.11).

Case 1: For the situation where $\lambda\notin\Lambda_2$, consider the case where the sliding mode has not yet entered the sliding domain. $sign(s(t_i))=sign(s(t))$ and $\|s(t_i)\|>\mu$. The derivative of $V_{(\xi)}$ is:

$$\dot{V}_{(\xi)}\le\frac{1}{\|\xi_1(t)\|}\begin{pmatrix}4\grave{o}K_2\xi_1^2(t)+(\theta+4\grave{o}^2-2k_2)\xi_1(t)\xi_2(t)\\ -2\grave{o}\xi_2^2(t)\xi_2(t)-(\theta+4\grave{o}^2)K_1\|\xi_1(t)\|\|\xi_1(t_i)\|\\ +2\grave{o}K_1\xi_2(t)\xi_1(t_i)+(\theta+4\grave{o}^2)\|\xi_1(t)\|\|L\|\|C\|\|e(t)\|\\ +2\grave{o}\|\xi_2(t)\|\|L\|\|C\|\|e(t)\|\end{pmatrix} \quad (3.20)$$

Given the existence of $\|s(t_i)\|=\|s(t_i)+C[x'(t_i)-x'(t)]\|$, we can get:

$$\|s(t)\|-\|C\|\|e(t)\|\le\|s(t_i)\|\le\|s(t)\|+\|C\|\|e(t)\| \quad (3.21)$$

According to equation (3.9), equation (3.21) can become:

$$\|\xi_1(t)\|-\sigma_2\le\|\xi_1(t_i)\|\le\|\xi_1(t)\|+\sigma_1 \quad (3.22)$$

where $\sigma_1=\frac{1}{2L}+(\frac{1}{4L^2}+\frac{\lambda_M}{L})^{\frac{1}{2}}$, $\sigma_2=\frac{1}{2L}+(\frac{\lambda_M}{L})^{\frac{1}{2}}$.

Define $\zeta(t)=[\|\xi_1(t)\| \ \|\xi_2(t)\|]^T$, according to (3.9), and given the existence of $\lambda\le\lambda_M$, we have:

$$\dot{V}_{(\xi)}\le-\frac{1}{\|\xi_1(t)\|}\zeta^T(t)Q\zeta(t)+\frac{M}{\|\xi_1(t)\|}\|\xi(t)\| \quad (3.23)$$

where $M=(\theta+4\grave{o}^2)(K_1^*\sigma_2+\sigma_1+\lambda_M)+2\grave{o}(K_1^*\sigma_1+\sigma_1+\lambda_M)$, the matrix Q is:

$$Q=\begin{bmatrix}(\theta+4\grave{o}^2)K_1-4\grave{o}K_2-(\theta+4\grave{o}^2) & *\\ -\frac{1}{2}(\theta+4\grave{o}^2-2K_2+2\grave{o}K_1+2\grave{o}) & 2\grave{o}\end{bmatrix} \quad (3.24)$$

To ensure that Q is positive definite, it must satisfy:

$$K_1>\frac{\theta+4\grave{o}^2+\grave{o}}{\theta}+\frac{1}{4\grave{o}\theta}(\theta+4\grave{o}^2+2\grave{o})^2 \quad (3.25)$$

According to equations (3.18) and (3.23), the derivative of the Lyapunov function (3.17) is:

$$\dot{V}_{(\xi,K)}\le-\Lambda V_{(\xi,K)}^{\frac{1}{2}}-\frac{1}{\|\xi_1(t)\|}\lambda_{\min}(Q)\vartheta\|\xi(t)\|^2+\frac{M}{\|\xi_1(t)\|}\|\xi(t)\| \quad (3.26)$$

Where $0\le\vartheta<1$, $\Lambda=\min(\bar{\eta},\omega_1\sqrt{l_1},\grave{o}\omega_1\sqrt{l_1})$, $\bar{\eta}=\lambda_{\min}(Q)(1-\vartheta)/\sqrt{\lambda_{\max}(P)}$.

According to equation (3.26), if $\|\xi(t)\|\ge M/\lambda_{\min}(Q)\vartheta$, then $\dot{V}_{(\xi,K)}\le-\Lambda\sqrt{V_{(\xi,K)}}$, the sliding variable $s(t)$ will converge within the domain $\Omega_1=\{\|s\|\le M^2/\lambda_{\min}^2(Q)\vartheta^2\}$.

The second scenario is when $\lambda\in\Lambda_2$, implying that the trajectory has already entered the sliding domain. We proceed with the analysis. The maximum deviation of the trajectory $s(t)$ within any time interval $t\in[t_i,t_{i+1})$ is given by:

$$\|s(t_i)-s(t)\|=\|Cx(t_i)-Cx(t)\|\le\|C\|\|e(t)\|\le\frac{\sigma\|s(t_i)\|^{\frac{1}{2}}}{L} \quad (3.27)$$

Therefore, the maximum boundary value of the sliding domain at this time is:

$$\Omega_2=\left\{\|s(t)\|\le\frac{\sigma\|s(t_i)\|^{\frac{1}{2}}}{L}\right\} \quad (3.28)$$

In conclusion, for all *t*, the sliding surface eventually converges within the domain (3.11), thus proving Theorem 1.

Step 2: Prove the stability of the closed-loop system after the sliding surface reaches the sliding domain. Construct the Lyapunov function $V_{\tilde{x}_1}=\frac{1}{2}\tilde{x}_1^T\tilde{x}_1$, and according to (3.2), taking its derivative, we can get:

$$\begin{aligned}\dot{V}_{\tilde{x}_1}&=\frac{1}{2}\tilde{x}_1^T\dot{\tilde{x}}_1+\frac{1}{2}\dot{\tilde{x}}_1^T\tilde{x}_1\\ &=\tilde{x}_1^T(s-\Lambda\tilde{x}_1-e_1^1)\le-\lambda_{\min}(\Lambda)\|\tilde{x}_1\|^2+\|\tilde{x}_1\|\|s\|+\|\tilde{x}_1\|\|e_1^1\|\end{aligned} \quad (3.29)$$

It can be observed that when $\lambda_{\min}(\Lambda)\|\tilde{x}_1\|>\|s\|+\|e_1^1\|$, $\dot{V}_{\tilde{x}_1}<0$. Therefore, it can be concluded that the error trajectory $\tilde{x}_1$ converges to:

$$\Sigma=\left\{\|\tilde{x}_1\|\le\max\{\frac{M^2}{\lambda_{\min}(\Lambda)\lambda_{\min}^2(Q)\vartheta^2},\frac{\sigma\|s(t_i)\|^{\frac{1}{2}}}{L\lambda_{\min}(\Lambda)}\}\right\} \quad (3.30)$$

At this point, the proof is complete.

Note 1: The control gains K_1 and K_2 are bounded. When $s(t)>\mu$, $K_1=K_1(0)+\omega\, t\sqrt{l_1/2}$, where $0\le t\le t_f$, this implies that $K_1(t)$ is bounded. Since $K_2=\grave{o}K_1$, this implies that K_2 is also bounded. When $s(t)\le\mu$, the control gains K_1 and K_2 are decreasing. Therefore, there exist positive numbers $K_1\le K_1^*$ and $K_2\le K_2^*$.

IV. INTER-EVENT TIME ANALYSIS

Let $T_i=t_{i+1}-t_i$ be defined as the inter-event time. To avoid the occurrence of Zeno behavior, the inter-event time in event-triggered systems must have a positive lower bound, as stated in the following theorem:

Theorem 2 Considering the system (3.3) and the controller (3.4), under the triggering condition (3.7), there exists a positive lower bound for the inter-event time between any two consecutive triggering sequences

Proof: letting $\Gamma=\{t\in[t_i,t_{i+1}):\|e(t)\|=0\}$, for all $t\in[t_i,t_{i+1})\backslash\Gamma$, we can get:

$$\frac{d}{dt}\|e(t)\|\le \mathrm{L}\|e(t)\|+K_1\|C^{-1}\|\|s(t_i)\|^{\frac{1}{2}}+K_2\|C^{-1}\|T_i \quad (4.1)$$

Integrating both sides of the equation yields:

$$\|e(t)\|\le\frac{K_1\|s(t_i)\|^{\frac{1}{2}}+K_2T_i}{\mathrm{L}\|C\|}(e^{L(t_{i+1}-t_i)}-1) \quad (4.2)$$

sAccording to (3.30), it can be inferred that the maximum increase of the error at time t is given by $Z = \max\{M^2/\lambda_{\min}(\Lambda)\lambda_{\min}^2(Q)\vartheta^2, \sigma\|s(t_i)\|^{\frac{1}{2}}/L\lambda_{\min}(\Lambda)\}$. Combining (4.2), one has that:

$$Z \le \frac{K_1\|s(t_i)\|^{\frac{1}{2}} + K_2 T_i}{\mathrm{L}\|C\|}(e^{L(t_{i+1}-t_i)} - 1) \tag{4.3}$$

Defining a function of T_i:

$$F(T_i) = \frac{K_1\|s(t_i)\|^{\frac{1}{2}} + K_2 T_i}{\mathrm{L}\|C\|}(e^{L(t_{i+1}-t_i)} - 1) - Z \tag{4.4}$$

It can be easily observed that for $T_i \ge 0$, this function is monotonically increasing. Since $F(0) = -Z < 0$, there exist values $r > 0$ such that $F(r) = 0$. Therefore, the solution to equation (4.3) is $T_i \ge r \ge 0$, and Theorem 2 is proved.

V. SIMULATION

MATLAB/Simulink software is utilized for simulation to demonstrate the effectiveness of the proposed method. The model parameters for the quadrotor are referenced from [17], and the parameter settings for the adaptive sliding mode attitude controller are shown in Table 1.

Table 1. Parameters of controller

Parameter	Λ	ω_{1i}	l_{1i}	μ_i	K_{mi}	η	ò
Value	$diag\{10,10,10\}$	6	2	0.05	0.8	0.8	0.1

The reference signals are selected as $\Theta_d = [0.1\times\cos(t), 0.1\times\sin(t), 0.1\times\sin(t)]^T$. The initial states are set $\Theta(0) = [0, 0.1, 0.1]^T$. The external disturbances are given by $d_1 = 0.05 sin(t)$, $d_2 = 0.05\cos(t)$. The parameters of the observer are chosen as $\lambda_0^1 = 10$, $\lambda_0^2 = 5$, $\lambda_1^1 = 10$, $\lambda_1^2 = 3$, $\lambda_2^1 = 1$, $L_1 = 1$, $L_2 = 1$. The dynamic parameter design for the triggering rule as: $\dot{\lambda} = -\lambda(t) - \lambda(t)s(t) + (|s(t)|^{\frac{1}{2}} - L\|C\|\|e(t)\|)|s(t)|$, and $\lambda_M = 5$, $L = 5$, $\sigma = 0.6$.

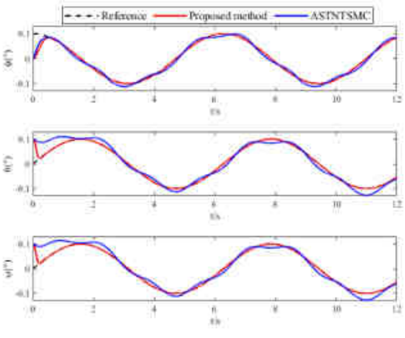

Fig. 1. Comparison of attitude tracking

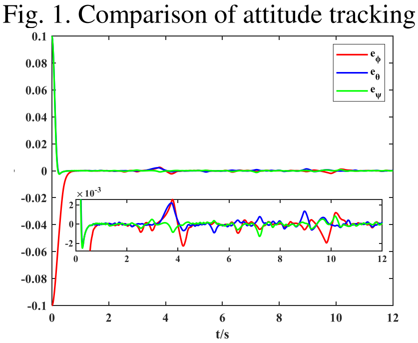

Fig. 2. Tracking errors of the attitude angles

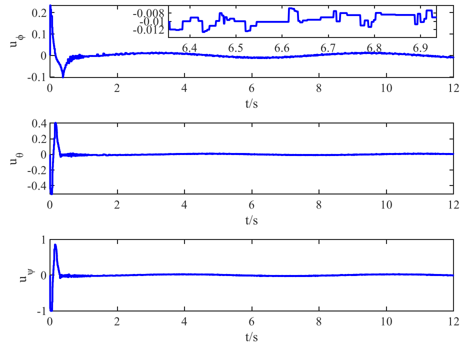

Fig. 3. Control signals

Fig. 1 presents a comparison of the tracking performance of the three attitude angles of a quadrotor between the proposed algorithm in this paper and the algorithm in [8]. In the presence of mismatched disturbances, the proposed approach demonstrates stable tracking of the three attitude angles, showing the significant robustness of the proposed algorithm. Fig. 2 illustrates the tracking errors of the three attitude angles, indicating that the control algorithm designed in this chapter achieves high attitude tracking accuracy. Fig. 3 displays the control inputs for the quadrotor's attitude control. Under the dynamic event-triggering mechanism, the controller is updated 976 times, significantly reducing computational resource waste compared to the traditional fixed-period update of 12001 times, thereby conserving system resources. Fig.4 represents the event-triggered times, with a lower bound existing between consecutive updates of the controller. This implies that the controller's update frequency is finite, indicating the absence of Zeno behavior in the system.

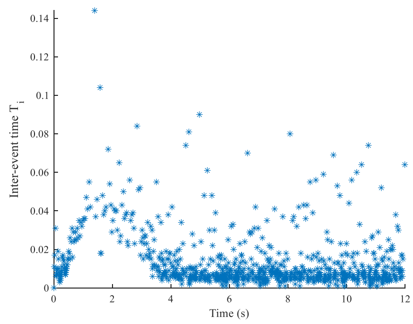

Fig. 4. Evolution of inter-event time

VI. CONCLUSION

This paper proposes an event-triggered adaptive super-twisting control method based on a disturbance observer, which effectively improves the robustness and performance of the system while optimizing resource utilization. The adaptive super-twisting control strategy based on the disturbance observer is employed to compensate for both matched and mismatched disturbances, enhancing the system's robustness and performance. Additionally, a dynamic event-triggering control strategy is introduced to optimize resource utilization. It allows for flexible determination of when to trigger the controller's update based on the actual state changes of the system, significantly reducing the number of control updates and conserving resources.

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