New efficient controllers' design method for first order systems, second order systems, FOPDT/SOPDT and systems that can be approximated as such

Farhan A. Salem

Abstract— This paper summarizes writer's previous works and proposes a new simple and efficient P-, PI-, PD- and PID-controllers design method, to cope with a wide range of systems; first order systems, second order systems, FOPDT/SOPDT and systems that can be approximated as such, the proposed method is based on relating and calculating controllers' parameters-gains based on plant's parameters. Simple set of formulas are proposed for calculating and soft tuning of controllers' gains, to achieve an important design compromise; acceptable stability, and medium fastness of response, the proposed method was tested and compared with world wide known and applied controllers design methods and using MATLAB/Simulink for different systems, the obtained results show simplicity and applicability of proposed design approach.

Index Terms— **Controller, Controller design.**

I. INTRODUCTION

The term control system design refers to the process of selecting feedback gains (poles and zeros) that meet design specifications in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and insight into the dynamics of the plant (Katsuhiko Ogata,1997)(Ahmad A. Mahfouz,et al,2013). An important compromise for control system design is to result in acceptable stability, and medium fastness of response, one definition of acceptable stability is when the undershoot that follows the first overshoot of the response is small, or barely observable. Beside world wide known and applied controllers design methods including Ziegler and Nichols known as the "process reaction curve" method (J. G. Ziegler,1943) and that of Cohen and Coon (G. H. Cohen,1953) Chiein-Hrones-Reswick (CHR), Wang–Juang–Chan, many controllers design methods have been proposed and can be found in different texts including; (Katsuhiko Ogata,1997)(Astrom K.J et al1994)(Ashish Tewar, et all,2002)(Norman S. Nise,2011)(Gene F. Franklin, et all, 2002)(Dale E. Seborg, et all, 2004)(Dingyu Xue,et all, 2004)(Chen C.L, 1998) (R. Matousek,2012)(K. J. Astrom,2001)(Susmita Das, et all,2012)(L. Ntogramatzidis, et all,2010)(M.Saranya, et all, 2012)(Fernando G. Martons,2005)(Saeed Tavakoli, et all, 2003)(Juan Shi , et all,2004)(Farhan A. Salem,2014), each method has its advantages, and limitations. (R. Matousek, 2012) presented multi-criterion optimization of PID controller by means of soft computing optimization method HC12. (K. J. Astrom, et all, 2001) Introduced an improved PID tuning approach using traditional Ziegler-Nichols tuning method with the help of simulation aspects and new built in function. (L. Ntogramatzidis, et all, 2010) a unified approach has been presented that enable the parameters of PID, PI and PD controllers (with corresponding approximations of the derivative action when needed) to be computed in finite terms given appropriate specifications expressed in terms of steady-state performance, phase/gain margins and gain crossover frequency. (M.Saranya, et all, 2012) proposed an IMC tuned PID controller method for the DC motor for robust operation. (Fernando G. Martons,2005) proposed a procedure for tuning PID controllers with Simulink and MATLAB. (Saeed Tavakoli, et all, 2003) presented using dimensional analysis and numerical optimization techniques, an optimal method for tuning PID controllers for FOPDT systems. (Juan Shi, et all, 2004) presented some derivation of IMC controllers and tuning procedures when they are applied to SOPDT processes for achieving set-point response and disturbance rejection tradeoff. (Farhan A. Salem,2013) proposed a new and simple controllers efficient model-based design method, based on relation controller's parameters and system's parameters. Many tuning formulas for PID controllers have been obtained for FOPDT processes (B.A. Ogunnaike, et al, 1994)(J.Shi, et al, 2002)(F.G. Shinskey, 1998), by optimizing some time-domain performance criteria. (Juan Shi, et all, 2004) proposed set point response and disturbance Rejection tradeoff for second-order plus dead time processes.(Jan Cvejn,2011) presented simple PI/PID controller tuning rules for FOPDT plants with guaranteed closed-loop stability margin.

This paper summarises writer's previous works (Farhan A. Salem,2013)(Farhan A. Salem,2014)(Farhan A. Salem,2014)(Farhan A. Salem,2014),, and proposes simple and easy P, PI, PD and PID controller design method, the proposed method is based on relating controller(s)' parameters and plant's parameters to result in meeting an important design compromise; acceptable stability, and medium fastness of response. To achieve smoother response in terms of minimum *PO%, 5T*, T_S, and E_{SS}, soft tuning parameters with recommended ranges are to be introduced. To achieve approximate desired output response or a good start design point, expression for calculating corresponding controller's gain are to be proposed.

1.1 Controllers Mathematical Modeling

The controllers that will be considered are P-, PD-, PI- and PID controllers; where P- term gives control system an instant response to an error, the I- term eliminates the error in the longer term, and D-term have the effect of reducing the maximum overshoot and making the system more stable

Farhan A. Salem Mechatronics Engineering prog. Dept. of Mechanical Engineering, college of Engineering, Taif University, 888, Taif, Saudi Arabia. [2]Alpha Center for Engineering Studies and Technology Researches (ACESATR), Amman, Jordan

by giving control system additional control action when the error changes consistently, it also makes the loop more stable (up to a point) which allows using a higher controller gain and a faster integral-terms (shorter integral time or higher integral gain)(Jacques F. Smuts, 2011) .
The transfer function of P-Controller is given by Eq.(1). The transfer function of PD-controller is given by Eq.(2). Equation (2) can be expressed in terms of derivative time constant T_D to have the manipulated shown form. The transfer function of PI-Controller is given by Eq.(3), Equation (3) can be expressed in terms of integral time constant T_I, to have the from given by (4). The transfer function of PID-controller is given by Eq.(5), this equation is second order system, with two zeros and one pole at origin, and can be expressed to have the form given by Eq.(6), which indicates that PI and PD controllers are special cases of the PID controller. The ability of PI and PID controllers to compensate many practical processes has led to their wide acceptance in many industrial applications. The transfer function of PID controller, also, can also be expressed to have the manipulated form given by Eq.(7), since PID transfer function is a second order system, it can be expressed in terms of damping ratio ζ and undamped natural frequency ω_n to have the form given by Eq.(8). The transfer function of PID control given by Eq.(8) can, also, be expressed in terms of derivative time T_D and integral time T_I to have the form given by Eq.(9), since in Eq. (9) the numerator has a higher degree than the denominator, the transfer function is not causal and can not be realized, therefore this PID controller is modified through the addition of a lag to the derivative term, to have the form given by Eq.(10), where: N: determines the gain K_{HF} of the PID controller in the high frequency range, the gain K_{HF} must be limited because measurement noise signal often contains high frequency components and its amplification should be limited. Usually, the divisor N is chosen in the range 2 to 20. All these controllers and their form are simulated in MATLAB/Simulink shown in Figure 2. Lead and lag Compensators are soft approximations of PD, PI-controllers respectively, and used to improve systems transient and steady state response by presenting additional poles and zeros to the system, the transfer function of compensator are given by Eq.(11), where lag compensator is soft approximations of PI, with $Z_o > P_o$, and Z_o small numbers *near* zero , $Z_o = K_I/K_P$. The smaller we make P_o, the better this controller approximates the PI controller. Lead compensator is soft approximations of PD, with Z < P, where the larger the value of P, the better the lead controller approximates PD control. PD-controller is approximated to lead controller as given by Eqs.(12)

$$G_p(s) = \frac{U(s)}{E(s)} = K_p \quad (1)$$

$$G_{PD}(s) = K_P + K_D s = K_D(s + \frac{K_P}{K_D}) = K_D(s + Z_{PD})$$

$$G_{PD}(s) = K_P + K_D s = K_P(1 + \frac{K_D}{K_P}s) \quad (2)$$

$$G_{PD}(s) = K_P(1 + T_D s) = K_P(1 + \frac{T_D s}{1 + T_D s}) = K_P(1 + \frac{T_D s}{1 + T_D s / N})$$

$$G_{PI}(s) = K_P + \frac{K_I}{s} = \frac{K_P s + K_I}{s} = \frac{K_P(s + \frac{K_I}{K_P})}{s} = \frac{K_P(s + Z_{PI})}{s} \quad (3)$$

$$G_{PI}(s) = K_P + \frac{K_I}{s} = K_P(1 + \frac{K_I}{K_P s}) = K_P(1 + \frac{1}{T_I s}) = K_P\left(\frac{T_I s + 1}{T_I s}\right) \quad (4)$$

$$u(t) = K_P(e(t) + \frac{1}{T_I}\int e(t)dt) \Rightarrow u(t) = K_P(e + \frac{e}{Ti})$$

$$G_{PID}(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} = \frac{K_D\left[s^2 + \frac{K_P}{K_D}s + \frac{K_I}{K_D}\right]}{s} \quad (5)$$

$$G_{PID} = \frac{K_D(s + Z_{PI})(s + Z_{PD})}{s} = K_D(s + Z_{PI})\frac{(s + Z_{PD})}{s} = G_{PD}(s)G_{PI}(s) \quad (6)$$

$$G_{PID} = \frac{K_D(s + Z_{PI})(s + Z_{PD})}{s} = \frac{K_D s^2 + (Z_{PI} + Z_{PD})K_D s + (Z_{PI}Z_{PD}K_D)}{s} \quad (7)$$

$$G_{PID} = \frac{K_D s^2}{s} + \frac{K_D s(Z_{PI} + Z_{PD})}{s} + \frac{K_D(Z_{PI}Z_{PD})}{s} = (Z_{PI} + Z_{PD})K_D + \frac{(Z_{PI}Z_{PD}K_D)}{s} + K_D s$$

$$G_{PID}(s) = \frac{K_D\left[s^2 + \frac{K_P}{K_D}s + \frac{K_I}{K_D}\right]}{s} = \frac{K_D\left[s^2 + 2\xi\omega_n s + \omega_n^2\right]}{s} \quad (8)$$

$$\text{where: } 2\xi\omega_n = \frac{K_P}{K_D}, \quad \omega_n^2 = \frac{K_I}{K_D}$$

$$G_{PID} = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) = K_P\frac{T_I T_D s^2 + T_I s + 1}{T_I s} \quad (9)$$

$$\text{where: } T_I = \frac{K_P}{K_I}, \text{Integral time}, T_D = \frac{K_D}{K_P}, \text{derivative time}.$$

$$G_{PID} = K_P\left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + \frac{T_D s}{N}}\right), \quad (10)$$

T_D/N - time constant of the added lag

$$G(s) = K_c\frac{(s + Z_o)}{(s + P_O)} \quad (11)$$

$$G_{Lead}(s) = K_P + K_D\frac{Ps}{s + P} = \frac{K_P(s + P) + K_D Ps}{s + P} = (K_P + K_D P)\frac{s + \left[\frac{K_P P}{K_P + K_D P}\right]}{s + P} = K_c\frac{(s + Z_o)}{(s + P_O)} \quad (12)$$

2. Plants' standard forms .

Most complex systems have dominant features that typically can be approximated based on plant's dominant poles approximation by either a first or second order system. Control system's response is largely dictated by those poles that are the closest to the imaginary axis, i.e. the poles that have the smallest real part magnitudes.
The general standard transfer function form of first order system's, and systems that can be approximated as first order systems, is given by Eq.(13), and can be represented by the block diagram shown in Figure 1(a), these systems are characterized, mainly, by time constant T. The general standard transfer function form of second order system's, and systems that can be approximated as second order systems is given by Eq.(14), and can be represented by the block

diagram shown in Figure 1(b), these systems are characterized, mainly, by damping ratio ζ and undamped natural frequency ω_n.

A large number of industrial plants can approximately be modeled by FOPTD/SOPTD process (Katsuhiko Ogata,1997),(Saeed Tavakoli,et all, 2003). FOPDT models are a combination of a first-order process model with dead-time, it can be represented by the block diagram shown in Figure 1(c), with transfer function given by Eq.(15) and response curve shown in Figure 1(e), this s-shape curve with no overshoot is called reaction curve, it is characterized by three parameters ; the delay time L, time constant T and steady state level K. A second order plus dead-time transfer function is given by Eq.(16), these are a combination of a second-order process model with dead-time, and can be represented by the block diagram shown in Figure 1(d).

$$G(s) = \frac{K_{DC}}{Ts+1} \quad (13)$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (14)$$

$$\frac{C(s)}{R(s)} = \frac{Ke^{-Ls}}{Ts+1} \quad (15)$$

$$\frac{C(s)}{R(s)} = \frac{Ke^{-Ls}}{(T_1 s+1)(T_2 s+1)} = \frac{Ke^{-Ls}}{(s^2/\omega_n^2)+(2\xi\, s/\omega_n)+1} = \frac{Ke^{-Ls}}{(\tau^2\omega_n^2)+(2\xi\tau\, s)+1} \quad (16)$$

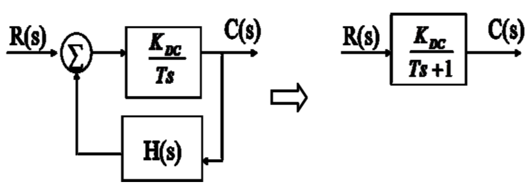

Figure 1(a) Block diagram representation of first order system

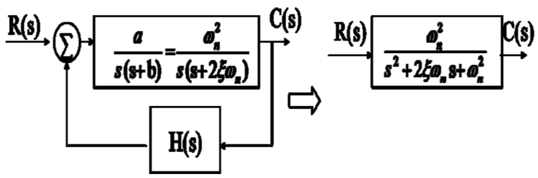

Figure 1(b)Block diagram representation of second order system

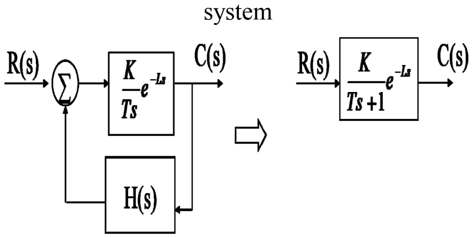

Figure 1(c) Block diagram representation of FOPDT process

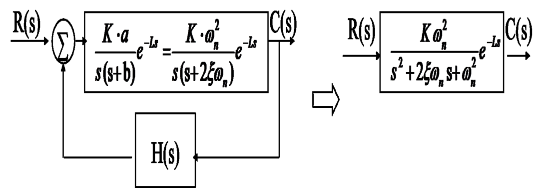

Figure 1((d) Block diagram representation of SOPDT process

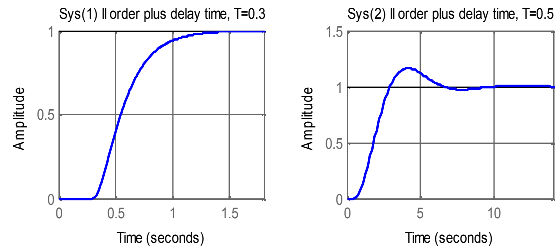

Figure 1(e) FOPTD/SOPTD S-shaped curve

3. Proposed Controllers design method.

The proposed (P, PI, PD, and PID) controllers design method is based on relating and selecting controllers' parameters based on process's parameters (e.g. ζ, ω_n T, K and L), as given by Eq.(17), these function to be reduced in terms of variables and based on required controller type and process order, to result in expressions for calculating controllers' parameters to meet acceptable stability, and medium fastness of response. The methods applied for derivation proposed expressions include dimensional analysis to simplify a problem by reducing the number of variables to the smallest number of essential ones (M. Zlokarni, 1991), mathematical modeling and solving step response for tracking acceptable acceptable stability and medium fastness of response in terms of minimum *PO%, 5T,* T_S, and E_{SS}, analysis of results based on relating processes parameters to controllers parameters, considering effects of each parameter on overall system response, e.g. relating controller's derivative time constant T_D to process's dead time L ,and integral time constant T_I to process's time constant T, and finally trial and error. for some cases the proposed method present a good starting point to get a process under control, to achieve desired and/or smoother response, soft tuning parameters (α, β and ε) with recommended ranges are to be introduced.

$$K_x = f_y(\zeta, \omega_n, T, K, L)$$

(17)

4. Proposed controller design expressions

4.1 Controller design for First order systems

The proposed design method and expressions for controllers terms selection and design for first order systems are summarized in Table 1 , where T : plant's time constant, R: desired output value (Reference input), α, β and ε: soft tuning parameters for controller's terms-gains, N: filter coefficient.

Table
1: P, PI ,PD, PID controllers terms for *first* order systems

Controller type	K_P		K_I	K_D	T_D	T_I	N
P-controller	αRT		0	0	0	0	0
For sys. With small DC gain and/or small T	$10\alpha RT$		0	0	0	0	0
For meeting desired specifications	$\alpha = \frac{T}{T_{desired}}$	αRT					

PD-Controller	$K_P = \alpha R T$	0	$K_D = T / \alpha$	$T_D = \frac{1}{R\alpha}$	0	$1 \div 22$
	$K_P = \alpha T$	0	T	$T_D = 1/\alpha$	0	
For sys. small DC gain and/or small T	$K_P = \alpha R^2 T$	0	$K_D = T / \alpha$	$T_D = 1/R^2\alpha$	0	

PI-Controller	$K_P = \alpha T$	$K_I = \frac{K_P}{T} = \frac{\alpha T}{T} = \alpha$	0	0	$T_I = T$	$1 \div 22$
For sys. small DC gain and/or small T	$K_P = \alpha T$	$K_I = \frac{\alpha}{T}$	0	0	$T_I = T^2$	
For meeting desired specifications	$\alpha = \frac{\psi * R * T}{Dc_{gain} * T_{desired}}$ $\psi = [0.3:1]$	$K_I = \frac{\alpha}{T}$	0	0	$T_I = T^2$	

First PID-Controller design method	$K_P = \beta * T$	$K_I = \alpha * T$,	$K_D = \varepsilon * T$	$T_D = \frac{K_D}{K_P} = \frac{\varepsilon * T}{T} = \varepsilon$	$T_I = \frac{K_P}{K_I} =$ $T_I = \frac{T}{\alpha * T} = \frac{1}{\alpha}$	1-22
Second PID-Controller design method	$K_P = \beta * T$	T	α / T	$T_D = 1/T^2$	$T_I = \alpha$	

4.1.1 Testing proposed design method considering fourth order plant

The fourth order plant with transfer function given by Eq.(18), can be approximated based on dominant poles approximation as first order system with one pole at -1, and DC gain of 1, to have the form given by Eq(19), therefore the proposed controller design method can be applied to this system as first order system, then the calculated controller's gains can be applied to original fourth order system.

$$G(s) = \frac{10000}{s^4 + 126s^3 + 2725s^2 + 12600^s + 10000} \quad (18)$$

$$G(s) = \frac{1}{s+1} \quad (19)$$

Ziegler-Nichols step response method is used to compare the proposed P, PI, PD and PID controller design, the calculated controllers' gains applying Ziegler-Nichols are shown in Table 2(a), and applying proposed method are shown in Table 2(b), the resulted response curves are shown in Figure 1, these response curves show an acceptable response compromise in terms of acceptable stability, and medium fastness of response, are achieved.

Table 2(a) Calculated controllers' gains applying Ziegler-Nichols method

Design Method		K_P	K_I	K_D
Ziegler-Nichols Step Response (K_{crit}=25.1, P_{crit}=0.6327)	**P**	12.55	0	0
	PI	11.295	23.8028	0
	PD	20.08	0	0.9925
	PID	11.1524	34.3786	0.9045
		15.06	39.6713	0.9925

Table 2(b) Calculated P, PI, PD controllers' gains applying proposed method

Design Method		**α**	K_P	K_I	K_D
Proposed method	**P**	**1**	10	0	0
	PI	**3**	3	3	0
		1	1	1	0
	PD	**1.2**	12		0.8333

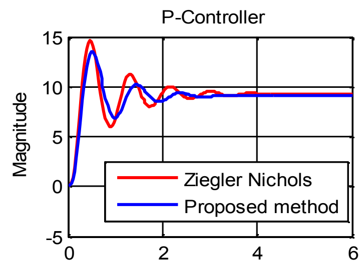

Figure 1(a) P-Controller design applying proposed and Ziegler-Nichols methods

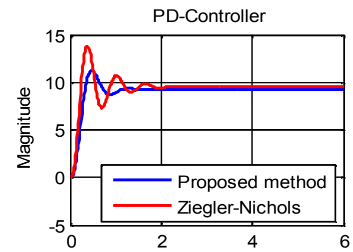

Figure 1(b) PD-Controller design applying proposed and Ziegler-Nichols methods

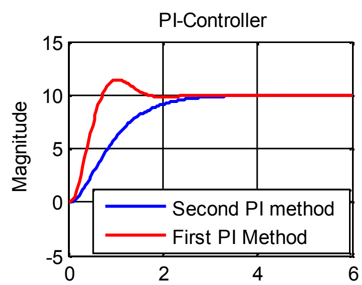

Figure 1(c) PI-Controller design applying both proposed methods

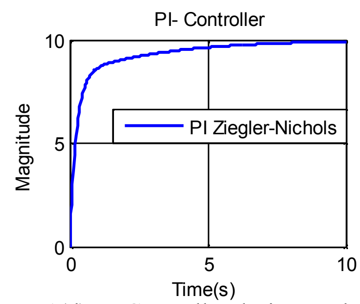

Figure 1(d) PI-Controller design applying Ziegler-Nichols

4.2 Controller design for Second order systems

The proposed design expressions for controllers terms selection and design for second order systems are summarized in Table 3 , where T : plant's time constant, R: desired output value (Reference input), ζ: damping ratio ,ω_n: undamped natural frequency , *α*, β and *ε*: soft tuning parameters for controller's gain's , N: filter coefficient.

Table 3: P, PI ,PD, PID controllers terms for *Second* order systems

Controller type	K_P	K_I	K_D	T_D	T_I	N	K_I
P-Controller	$\frac{\alpha R \omega_n}{\xi}$, $\alpha > 0$	0	0	0	0	0	0
For sys. small DC gain and/or small T	$\frac{\alpha R^2 \omega_n}{\xi}$, α *tuned from* 5.	0	0	0	0	0	0
To achieve desired output response in terms of output percent overshoot PO%	$K_P = 300\frac{\xi^2}{PO^2}$ $, For\ PO\% > 0.5$ $K_P = 100\frac{\xi^2}{\xi^2_{desired}}$ $, For\ PO\% < 0.5$						

PD-controller	K_P	K_I	K_D	T_D	T_I	N
PD-controller	$K_P = \frac{\alpha R}{\xi \omega_n}$		$K_D = 2.9 \alpha R \xi \omega_n$	$T_D = \frac{K_D}{K_P} =$ $=2.9\xi^2\omega_n^2$	0	1:22
Another more simple expressions	$K_P = 4\alpha V_{in}$		$K_D = 3\alpha V_{in}$	$T_D = \frac{K_D}{K_P} = 0.75$	0	1:22

For systems with *small T and/or small* DC gain	$K_P = \frac{\alpha R^2}{\xi \omega_n}$	0	$K_D = \alpha R^2 \xi \omega_n$	$T_D = \xi^2 \omega_n^2$	0	1:22
For systems *not* with small DC gain and /or small time constant,**T**o achieve desired response in terms of desired output percent overshoot PO%,	$K_P = 10\alpha V_{in}$		$K_D = \frac{3 * \xi_{PO\%}}{\alpha * \xi}$	$T_D = \frac{1}{10\xi * \xi_{PO\%} V_{in}}$	0	1:22
For systems with small DC gain and /or small time constant, To approximately achieve desired response in terms of desired output percent overshoot PO%	$K_P = \frac{0.5\alpha * (\xi + \omega_n)}{(\xi * \omega_n * \xi_{PO\%})}$		$K_D = \frac{[0.5:2]}{\xi * \alpha * \xi_{PO\%}}$	$T_D = K_D / K_P$		
PI-controller	***K_P***	***K_I***	***K_D***	***T_D***	***T_I***	***N***
	$K_P = 2.1\xi K_I$	$K_I = \frac{\xi + \omega_n}{7.5\xi \omega_n}$	*0*	*0*	$T_I = \frac{K_P}{K_I}$	*0*
For systems *with* small *DC gain* and/or small *time constant*	$K_P = \frac{15.1\alpha}{K_I}$	$K_I = \frac{\alpha(\xi + \omega_n)}{0.9\xi \omega_n}$	*0*	*0*	$T_I = \frac{K_P}{K_I}$	*0*
PID-controller	***K_P***	***K_I***	***K_D***	***T_D***	***T_I***	***N***
	$K_P = 1$	$K_I = \frac{\omega_n}{2\xi}$	$K_D = \frac{1}{2\xi \omega_n}$	$T_D = K_D$	$T_I = \frac{2\xi}{\omega_n}$	
Tuning limits	0:inf	$\varepsilon \frac{\omega_n}{2\xi}, \ \varepsilon = 0.1$	$\alpha \frac{1}{2\xi \omega_n}, \ \alpha = 0.5$	$\frac{\alpha}{2\xi \omega_n}$	$\frac{2\xi}{\varepsilon \omega_n}$	

4.2.1 Testing proposed design method

Considering a third order plant with transfer function given by Eq.(20), to verify proposed design, it will be compared with Ziegler-Nichols design method. This third order system, can be approximated as second order system with two repeated pole *P=1*, with *ζ=1, ω =1*. Designing P, PI, PD and PID controller applying Ziegler-Nichols design and proposed design methods will result in gains values listed in table 14, and the response curves shown in Figure 2, these response curves show that the proposed method is more simpler than Ziegler-Nichols, as well as a more smooth response with minimum overshoot and acceptable settling time are achieved.

$$G(s) = \frac{1}{(s+1)^3} \tag{20}$$

Table 3: controllers' gains values applying both Ziegler-Nichols and proposed design methods.

	Parameters		Pro.	PD		PI		PID		
Proposed method	**Test(1)**	**ζ**	**ω**	**α**	**ε**	**K_P**	**K_I**	**K_D**	**T_D**	**T_I**
	Test(1)	1	1	1	1	1	1	1	0.5	2
	Test(2)			1.5	1.1	0.3	0.55	0.75	0.75	1.8182

Ziegler-Nicols	**K_{crit}**	**P_{crit}**	**K_P**	**K_P**	**T_I**	**T_D**
	8	3.6	4.8	4.8	1.8	0.45

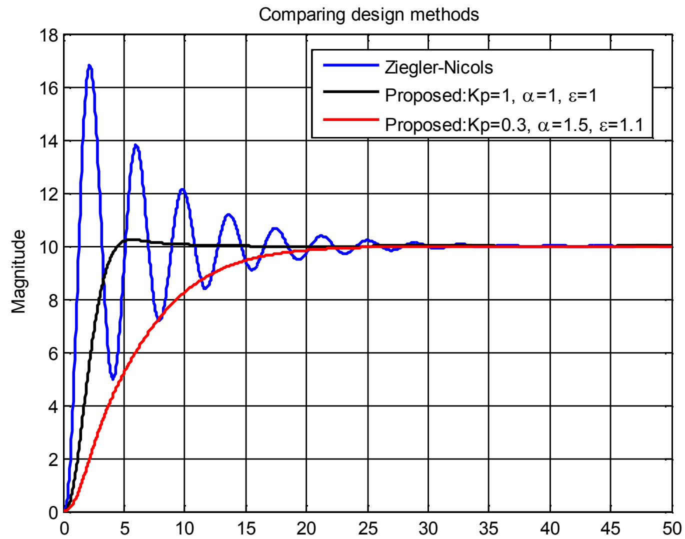

Figure 2 PID-controller design applying Ziegler-Nicols and proposed design methods

4.2.2 Testing the proposed PID design method for fourth order plant with transfer function given by Eq.(21), applying three different PID controller design methods, particularly, Ziegler Nichols frequency response, Ziegler-Nichols step response, and Chein-Hrones-Reswick design methods, will result in PID gains shown in Table 4(Robert A. Paz, 2001), , as shown in this table different values of PID gain are obtained and correspondingly different system's responses (see Figure 3), when subjected to step input of 10. Comparing shown response curves, show that the Chein-Hrones-Reswick design is, with less overshoot and oscillation (than Ziegler-Nicols), all three method almost, result in the same settling time.

Applying proposed PID-controller design method, based on plant's dominant two poles approximation, will result in smooth response curve without overshoot, and minimum zero steady state error, shown in Figure 3.

$$G(s) = \frac{10000}{s^4 + 126s^3 + 2725s^2 + 12600s + 10000} \qquad (21)$$

Table 4: PID controller design applying different control methods

Design Method	***K_P***	***K_I***	***K_D***
Ziegler Nichols Frequency Response	14.496	45.300	1.1597
Ziegler-Nichols Step Response	11.1524	34.3786	0.9045
Chein-Hrones-Reswick	5.5762	5.0794	0.4522
Proposed method	25	0.8632	0.1231
	ζ=3.0677 ω_n= 2.6481	α=90 ε= 4.9	

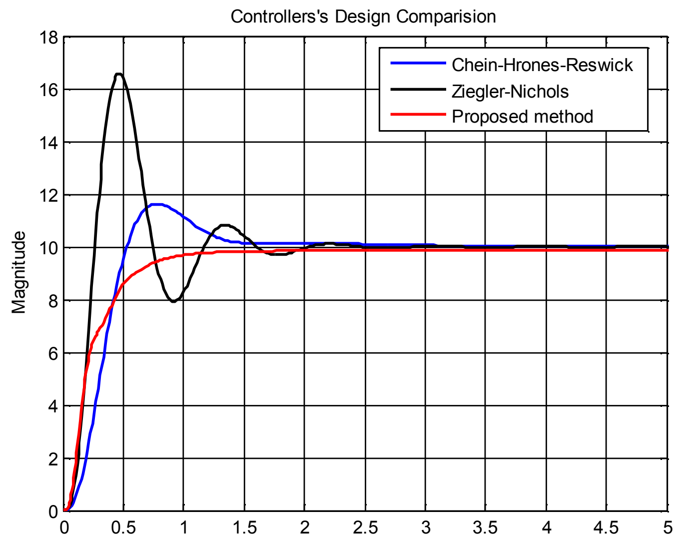

Figure 3 Step response curves obtained applying different design methodologies

3.3 Controller design for for FOPDT/SOPDT

3.3.1 Controller design for for FOPDT

The proposed design expressions for controllers terms selection and design for FOPDT are summarized in Table 5

,where : L the delay time , T: time constant and K: steady state level, ζ: damping ratio ,ω_n: undamped natural frequency , α, β and ε: soft tuning parameters for controller's gain's , N: filter coefficient.

Table 5: P, PI ,PD, PID controllers terms for FOPDT plants

The below expressions are applied first assigning α=1, and then parameter α can be tuned to soften resulted response. creasing α will result in increasing overshoot and reducing steady state error, also, assigning α big values may result in undamped response up to unstable response.

P-controller	K_P	K_I	K_D	T_D	T_I	N
	$K_P = \frac{T \cdot L}{K}$	0	0	0	0	0
	$K_P = \frac{\alpha \cdot T \cdot L}{K}$	0	0	0	0	0
For systems with *small T and/or small* DC gain	$K_P = \frac{10 \cdot T \cdot L}{K}$	0	0	0	0	0
	$K_P = \frac{10 \cdot \alpha \cdot T \cdot L}{K}$	0	0	0	0	0

PD-controller	K_P	K_I	K_D	T_D	T_I	N
	$K_P = 3.5\alpha \frac{T \cdot L}{K}$	0	$K_D = 0.9 \frac{T \cdot L}{K}$	$T_D = \frac{K_D}{K_P} = 0.2$	0	1:22
For processes with *small* DC gain *and/or small* time constant	$K_P = \alpha \frac{T \cdot L}{10K}$	0	$K_D = 0.2 \frac{T \cdot L}{K}$	$T_D = \frac{0.02}{\alpha}$	0	1:22
or both types of systems	$K_P = \alpha \frac{T}{K \cdot L}$		$K_D = K_P * T_D = 0.5 \frac{T}{K}$	$T_D = 0.5 \cdot L$	0	1:22

PI-controller	K_P	K_I	K_D	T_D	T_I	N
	$K_P = \alpha \cdot \frac{1.5K}{T \cdot L}$	$K_I = \frac{1.5L}{T}$	0	0	$T_I = \frac{K_P}{K_I}$	0

For processes with *small* DC gain *and/or small* time constant	$K_P = \alpha \cdot \frac{15K}{T \cdot L}$	$K_I = \beta \frac{K_P \cdot K}{L}$	0	0	$T_I = \frac{K_P}{K_I}$	0
applied for *most* types of FOPDT processes	$K_P = 0.3 \frac{T}{K \cdot L}$	$K_I = K_P / T_I$	0	0	$T_I = T$	0
PID-controller	K_P	K_I	K_D	T_D	T_I	**N**
	$K_P = 0.5\alpha TL$	$K_I = 0.5\beta LT$	$K_D = 0.$		$\frac{K_P}{K_I}$	2-20
The expressions are applied first assigning parameter *α=β=1*, and then parameters α and β can be tuned where, increasing β will speed up response and increase overshoot, meanwhile, increasing α will increase overshoot and slow response, the divisor *N* is chosen in the range 2 to 20.						

3.3.2 Testing proposed expressions for FOPDT

Testing proposed PID expressions for systems(a)(b)(c), given by Eq.(22), will result in response curves shown in Figure 4, the calculated gains and response measures are shown in table 6.

$$G_a(s) = \frac{1}{s+1} e^{-0.3s} \qquad G_b(s) = \frac{5}{10s+1} e^{-0.5s} ,$$
$$G_c(s) = \frac{0.005}{5s+1} e^{-0.9s} \qquad G_d(s) = \frac{0.1}{2s+10} e^{-0.2s}$$
$$G_{sys1}(s) = \frac{50e^{-0.3s}}{(0.1s+1)(0.2s+1)} = \frac{50e^{-0.3s}}{s^2+15s+50} \tag{22}$$
$$G_{sys2}(s) = \frac{e^{-0.3s}}{s^2+s+1} , \qquad G_{sys3}(s) = \frac{0.05}{2s^2+9s+1} e^{-0.5s}$$

Table 6: PID-Controller for systems(a)(b)(c) applying proposed and Z-N tuning method.

P-Controller		α	β	T	K	L	K_p	K_I	K_D	T_I	T_D	Mp	5T	DC gain
System(1)	Proposed method Eq.(24)	1	1	1	1	0.3	0.15	0.15	0.0333	1	0.2222	-	34	10
		1	5				0.15	0.75	0.0333	0.2	0.2222	1.4	15	10
		5	8				0.75	1.2	0.0333	0.625	0.0444	0.9	5.3	10
Ziegler-Nichols		-	-				4	6.67	0.6	0.6	0.15	5	6	10
System(2)	Proposed method	1	1	10	5	0.3	2.5	1.25	0.02	2	0.008	4.6	5	10
		2	0.5				5	0.63	0.02	8	0.004	6.3	13	10
		0.5	0.1				1.25	0.13	0.02	10	0.016	-	3.6	10
Ziegler- Nichols		-	-				24	24	6	1	0.25	See figure6(b)		
System(3) (*small DC*) Eq.(24)	Pro. method	1	1	5	0.05	0.9	2.25	0.02	0.0111	100	0.0049	2.6	75	10
		1	200				2.25	4.5	0.0111	0.5	0.0049	1.4	42	10
		1	150				2.25	2.25	0.0111	1	0.0049	1.4	50	9.8
Ziegler- Nichols		-	-				6.67	3.70	3	1.8	0.45	0.96	45	10

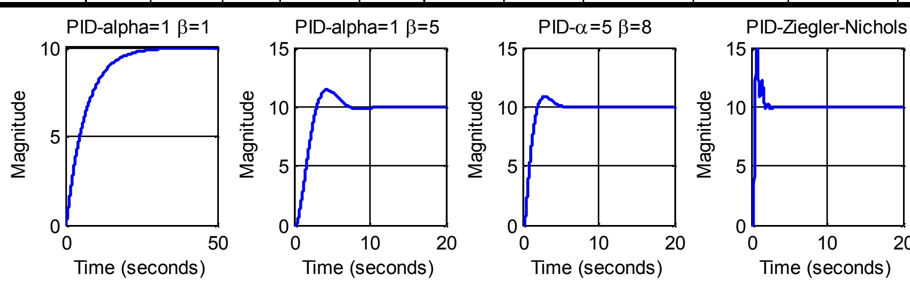

Figure 4(a) PID design for system(a)

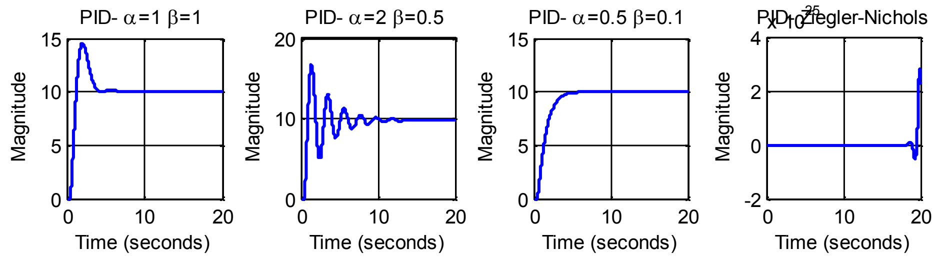

Figure 4(b) PID design for system (b)

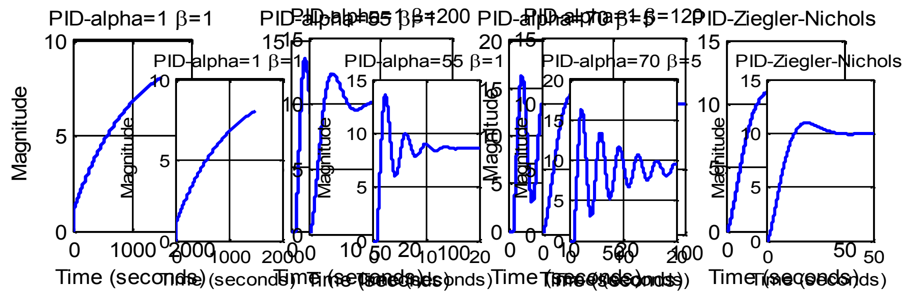

Figure 4(c) PID design for system (c)

3.3.3 Expressions for SOPDT

The proposed design expressions for controllers terms selection and design for SOPDT are summarized in Table 7

where : L the delay time , T: time constant and K: steady state level, ζ: damping ratio ,ω_n: undamped natural frequency , α, β and ε: soft tuning parameters for controller's gain's , N: filter coefficient.

Table 7: P, PI ,PD, PID controllers terms for SOPDT plants

P-controller	**K_P**	**K_I**	**K_D**	**T_D**	**T_I**	**N**
	$K_P = \frac{\alpha \cdot T}{K}$	0	0	0	0	0
	$K_P = \frac{\alpha \cdot T \cdot L \cdot R \cdot \omega_n}{\xi \cdot K}$	0	0	0	0	0
The expressions are applied first assigning parameter α=1, and then parameter α can be tuned to soften resulted response						

PI-controller	**K_P**	**K_I**	**K_D**	**T_D**	**T_I**	**N**
	$K_P = \alpha \frac{0.01}{K_I \cdot \xi \cdot T \cdot L}$	$K_I = \alpha \frac{(\xi + \omega_n)}{(\xi \cdot \omega_n)}$	0	0	$T_I = \frac{K_P}{K_I}$	0
For processes with *small* DC gain *and/or small* time constant	α can be assigned a small decimal value less than one (e.g. 0.1, 0.01), with limits for increasing α, since it may lead to instability.		0	0	$T_I = \frac{K_P}{K_I}$	0
applied for *most* types of FOPDT processes	$K_P = 0.3\frac{T}{K \cdot L}$	$K_I = K_P / T_I$	0	0	$T_I = T$	0
applied for most types of SOPDT processes,	$K_P = 0.3\frac{T}{K \cdot L}$	$K_I = T$	0	0	$T_I = \frac{K_P}{K_I}$	0
	$K_P = 0.3\frac{T\,\xi}{K \cdot L}$	$K_I = T$	0	0	$T_I = \frac{K_P}{K_I}$	0

PID-controller	**K_P**	**K_I**	**K_D**	**T_D**	**T_I**	**N**
	α	$\beta \frac{L \omega_n}{2\xi}$	$\varepsilon \frac{L}{2\xi\omega_n}$			2-20

For systems with small DC gain and/or small time constant	α	$K_I = \beta L \omega_n \xi$	$\varepsilon \dfrac{L}{2\xi\omega_n}$		$\dfrac{K_P}{K_I}$	
The expressions are applied first assigning parameter α=β=1, and then parameters can be tuned to soften resulted response. Where increasing alpha will reduce overshoot and speeding up response						

3.3.4 Testing proposed expressions for SOPDT

Testing proposed expressions for PID controller design, for systems(1)(2)(3) given by Eq.(22), will result in response curves shown in Figure 5, the calculated gains and response measures are shown in table 8.

Table 8: PID-Controller design for systems(1)(2)(3) applying proposed and Z-N tuning methods

P-Controller		α	β	ε	ζ	ωn	K	L	K_p	K_I	K_D	Mp	5T	DC gain
System(1)	Proposed method	1	1	1	1.0607	7.071	1	0.3	1	1.5	0.03	3.37	12	9.8
		0.5	1	1					0.5	1.5	0.03	-	3	10
		0.1	0.1	0.1					0.1	0.15	0.003	-	30	10
Ziegler-Nichols		-	-	-					0.8	1.333	0.12	0.8	6.2	10
System(2)	Proposed method	1	1	1	0.5	1	1	0.5	1	0.5	0.5	0.7	12	10
		1	1	2.5					1	0.5	1.25	0.03	11.5	10
		1.1	1	2.8					1.1	0.5	1.4	-	7	10
Ziegler- Nichols		-	-	-					4.8	4.8	1.2	See figure 7(b)		
System(3)	Pro. method	1	1	1	3.182	0.707	0.05	0.5	1	0.056	0.111	4.6	42	9.9
		250	1	1					300	1.125	0.11	3.3	11	9.35
		0.1	250	400	-				250	13.89	27.78	2.6		
Ziegler- Nichols			-	-						1.067	1.067	0.267	0.5	100

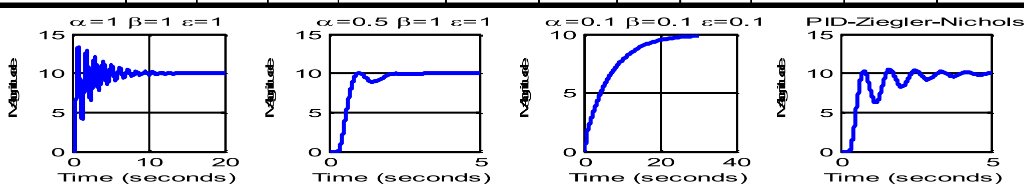

Figure 5(a) PID design for system (1)

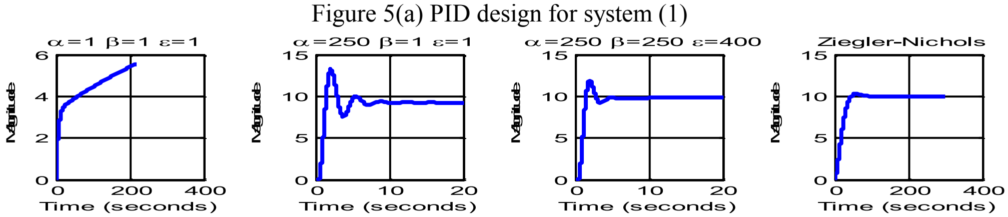

Figure 5(b) PID design for system (2)

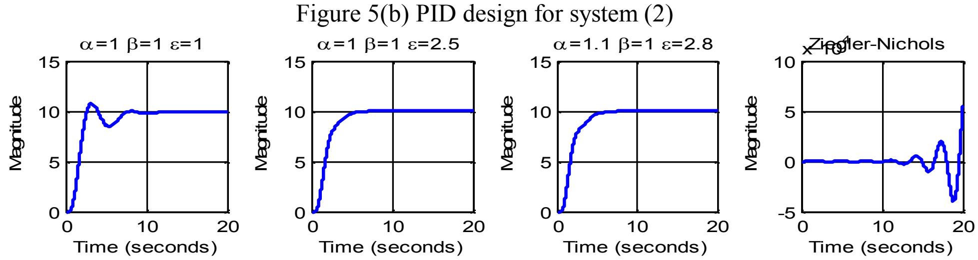

Figure 5(c) PID design for system (3)

Conclusion

A new simple and efficient time domain P, PI ,PD and PID controllers design method is proposed, to cope with a wide range of systems, to achieve an important design compromise; acceptable stability, and medium fastness of response, the proposed method is based on selecting controllers' parameters based on plant's parameters.

To achieve smoother response in terms of minimum *PO%*, *5T*, T_S, and E_{SS}, soft tuning parameters with recommended ranges are introduced. To achieve approximate desired output response or a good start design point, expression for calculating corresponding controller's gains are proposed. The proposed controllers design method was test for different system using MATLAB/Simulink software, the result obtained show applicability and simplicity of proposed design expressions.

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Farhan A. Salem : Now with Taif University, college of engineering , Dept. of Mechanical engineering, Mechatronics Engineering Program